Stability improvement for feedback noise control in ducts using a time delay compensator

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Abstract

When applying feedback control in active noise control, phase lag caused by transducer dynamics and time delay between microphones and noise sources limits the bandwidth of the controller considerably. The proposed method to increase stability is to compensate the phase lag by applying a Smith compensator. The compensator needs a model of the acoustic plant. As model, only the transducer dynamics are taken into account in this investigation. The compensator results in a considerable higher control bandwidth. Nevertheless, the demands on the transducer dynamics still remains high. The experiments demonstrate that phase lag can be partly compensated, but the poor transducer dynamics do not allow to close the control loop.

1. Introduction

In active noise control, the application of feedback controllers is not straight forward. One of the most important causes of this impopularity is their limited bandwidth, as consequence of the time delay between sensor and actuator, the presents of a high number of acoustical modes and the dynamics of the sensor and actuator. Feedback controllers put much higher demands on the dynamics of the transducers as the adaptive feedforward controllers. Otherwise, the controller itself can be implemented on cheap analog circuitry, while the feedforward algorithms has to be implemented digitally on DSP's. Even today, the mass production of active noise control systems is prevented mainly by the cost of the DSP-chip.

In modern control theory, the system dynamics are described in state space models [1]. The controller is developed by minimizing a quadratic cost criterion wherein the controller performance and the control effort are taken into account. It results in a state feedback controller. Two major problems comes forward in this approach. First, the state space model must contain sufficient dynamics to model the acoustic plant high enough in frequency. In practice, the plant dynamics have to be modeled one decade in frequency above the controller bandwidth, such that resonance amplitudes remain below the zero dB level of the open loop gain of the controlled system. Otherwise, instability can occur. Second, it is difficult to measure the state variables. When modal decomposition of the state space matrix is used, it leads to a simple diagonal system matrix. However, the state variables are not physical measurable quantities. Zhen Wu et al [2] uses the wave equation to build the state space model, and measures pressures and their derivatives at discrete positions. The total number of measurements equals the half of the number of states, which still makes it impractical to implement.

Clark and Frampton [3] presents a classical controller design approach using a lead-lag network to create a time delay compensation. The controller is based on the work of Clark and Cole [4], who have demonstrated that a colocated volume velocity source and pressure sensor can be used to attenuate the acoustic sound field using direct output feedback control. The controller can have infinite gain margins theoretically. In practice, the transducer dynamics and the acoustic time delay in particular limits the control bandwidth considerably. The applied lead-lag network compensates a part of the phase delay to obtain a higher control bandwidth. The theoretical maximum phase compensation for a first order lead-lag network amounts 90^o.

The time delay compensator proposed in this paper is based on a Smith compensator [5], which uses a plant model in conjunction with a pure time delay in feedback over the controller. The stability condition of this feedback loop around the controller determines the maximum time delay which can be compensated. The Smith compensator needs a plant model for time delay compensation. It is impractical to use the complete acoustic plant transfer function. The plant model has to be reduced to a model which remains robust against environmental changes which occurs during the controller action. In practice, it is possible to limit the plant model to the sensor and actuator dynamics, which can be developed and built to remain robust against environmental changes, even in different circumstances.

2. Smith compensator

The Smith compensator is applied in systems where the time delay becomes comparable with the plant time constant. The time delay continuously adds phase lag and reduces the stability margin of the controller. The stability margin can be recovered by connecting the Smith compensator in closed loop around the controller, as presented in figure 1.

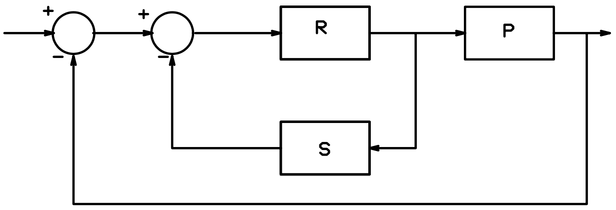

figure 1: Block diagram of a Smith compensated control system.

In this diagram, R is the controller transfer function, P is the plant transfer function and S is the Smith compensator. The plant transfer function contains a linear part P_0 and the plant time delay T_D.

$$P = P_0 \, \mathrm{e}^{-T_D \, \mathbf{s}} \tag{1}$$

wherein $\mathbf{s} = \mathbf{j}\,\omega$ is the Laplace variable. To demonstrate the effect of the compensator, the uncompensated controlled system becomes unstable when the open loop transfer function

$$R \, P_0 \, \mathrm{e}^{-T_D \, \mathbf{s}} = -1 \tag{2}$$

When the controller is compensated with a Smith compensator containing

$$S = P_0 \, (1 - \mathrm{e}^{-T_D \, \mathbf{s}}) \tag{3}$$

the stability condition of the compensated loop becomes

$$\frac{R \, P_0 \, \mathrm{e}^{-T_D \, \mathbf{s}}}{1 + R \, P_0 \, (1 - \mathrm{e}^{-T_D \, \mathbf{s}})} = -1 \tag{4}$$

which ultimately results in

$$R \, P_0 = -1 \tag{5}$$

The stability of the loop is now recovered, the time delay does not effect the stability. The compensator needs a model of the plant, which will determine the quality of the compensation. Also, an additional condition is, that the closed loop formed by the controller and the Smith compensator must be stable.

3. Smith compensator applied on active noise control in a duct

A feedback noise attenuator will be implemented on a duct, presented in figure 2. The duct is 2.480 m long and has 18.15 10^{-3} m^2 cross-section, wherein two 6” loudspeakers are fitted. The primary loudspeaker is situated at the right end of the duct, the secondary loudspeaker at 0.430 m from the open end. The error microphone is situated at 0.21 m from the open end.

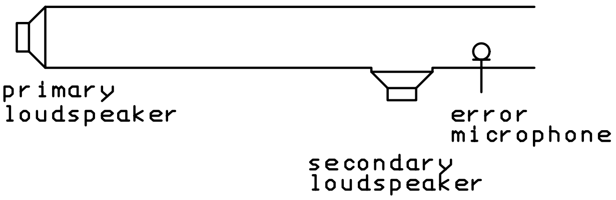

figure 2: Scheme of the duct system.

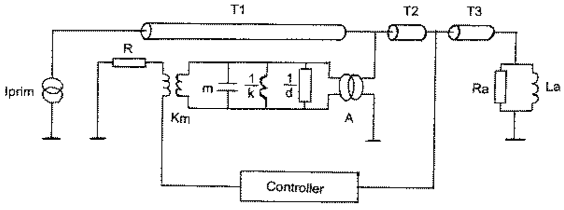

figure 3: Electrical analog circuit of the duct system.

A model is created using an electrical analog circuit, presented in figure 3. The noise generating loudspeaker is regarded as a volume velocity source $Iprim$. The duct is modeled as the sequence of the lossy transmission lines $T1$, $T2$ and $T3$. At the open end, the acoustic radiation impedance of free air, represented by the coil–resistor combination La and Ra, closes the circuit. The noise cancellation loudspeaker is modeled in more detail. The coil impedance R, the magnetic motor Km, the diaphragm mass m with suspension spring k, the damping d and finally, the acoustic radiation surface A are taken into account in the loudspeaker model. The higher diaphragm modes are neglected. The error microphone is modeled as an ideally signal amplifier. The controller contains a proportional controller with gain 10^4 and the Smith compensator.

The uncompensated plant transfer function is presented in figure 4. When the noise cancellation loudspeaker would be an ideally volume velocity source, the phase of the plant transfer function would be kept within 180^0, and the controller would be unconditionally stable, as demonstrated by Clark and Cole [4]. The loudspeaker dynamics and the time delay between loudspeaker and error microphone bound the control bandwidth.

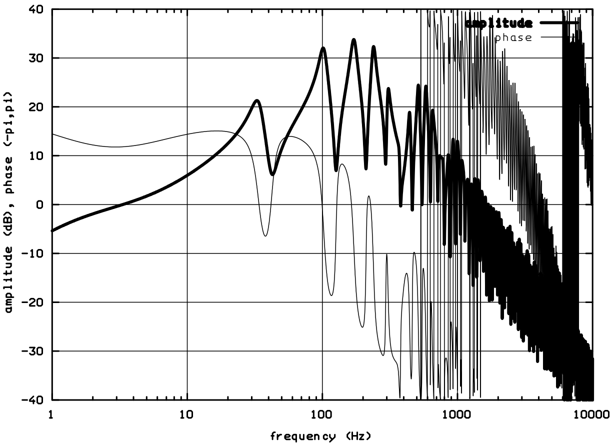

figure 4: Simulated open loop transfer function between error microphone and canceling loudspeaker in the duct.

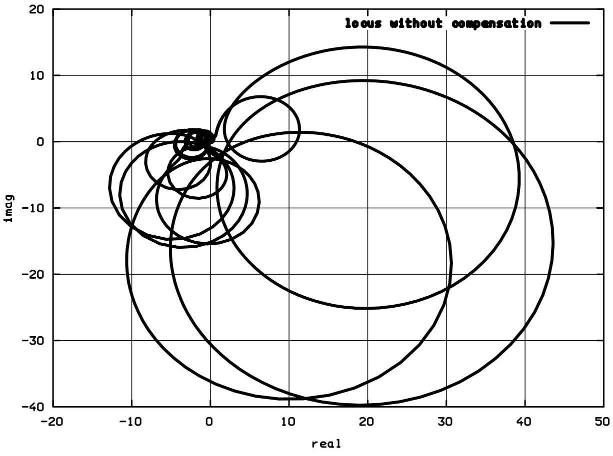

figure 5: Nyquist contour of the transfer function presented in figure 4.

The Nyquist plot, presented in figure 5, of this transfer function shows the circles of the resonances. Each circle is rotated by an angle dependent on the time delay. The -1-point is enclosed by the circles, consequently the closed loop is unstable.

Now, the Smith compensator is connected in closed loop around the controller. The Smith compensator contains the loudspeaker transfer function between output volume velocity and input current, and the time delay between loudspeaker and error microphone:

$$S = K_m \, A \, \frac{\mathbf{s}}{m \, \mathbf{s}^2 + d \, \mathbf{s} + k} \, (1 - \mathrm{e}^{-T_D \, \mathbf{s}}) \qquad (6)$$

wherein K_m the motor constant, A the sound radiating surface, m the diaphragm mass, k the diaphragm suspension spring, d the damping and T_D the time delay between loudspeaker and microphone. The controller transfer function with the compensator, presented in figure 6, is the closed loop transfer function between controller and compensator.

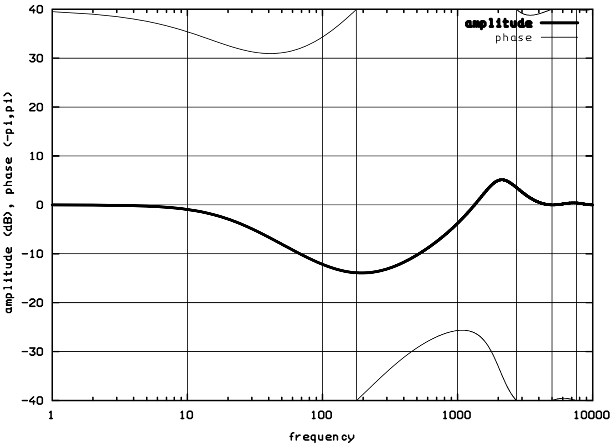

figure 6: Closed loop transfer function of the controller with the Smith compensator in feedback.

This compensator has the property that it has the same gain at the high frequency range in comparison with the low frequency range. The gain drops in between, where the phase lead is gained. A normal lead-lag circuit ends with a higher gain at the high frequency range in comparison with the low frequency range, depending on the gained phase lead.

This closed loop has to be stable. This can be judged in the Nyquist plot of the open loop transfer function of the controller with the compensator, presented in figure 7. The contour may not include the -1 instability point.

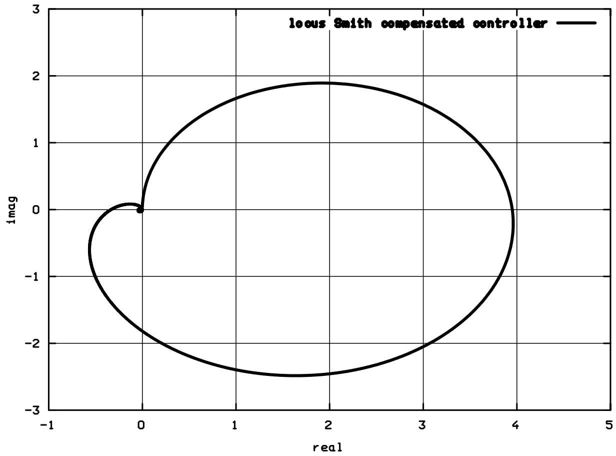

figure 7: Nyquist contour of the open loop transfer function of the controller with the Smith compensator.

The proportional controller using the Smith compensator is applied to cancel the noise radiating from the duct. The open loop transfer function of the duct with controller and compensator included is displayed in figure 8. The phase of the compensated system is kept within 180^o in the frequency range up to 2 kHz. The bandwidth can now be chosen around 800 Hz, while the uncompensated case is hardly controllable at all. The amplitude of the transfer function is decreased in the bandwidth region by the compensated controller, resulting in a slightly decrease of the control performance.

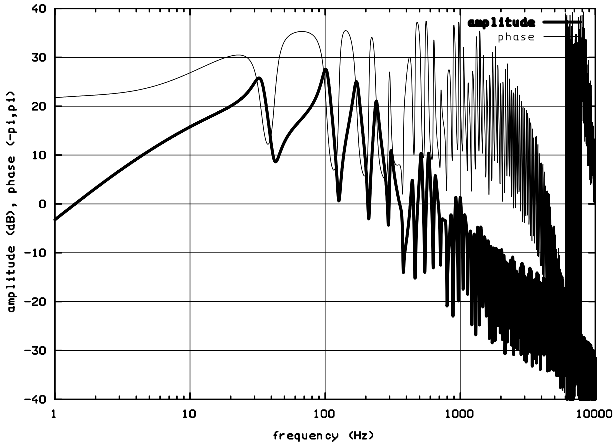

figure 8: Simulated open loop transfer function between output volume velocity of the loudspeaker and input sound pressure of the microphone, with proportional controller and Smith compensator.

The Nyquist loop of the controlled system is presented in figure 9. The instability point -1 is now excluded from the loop, resulting in a stable controller.

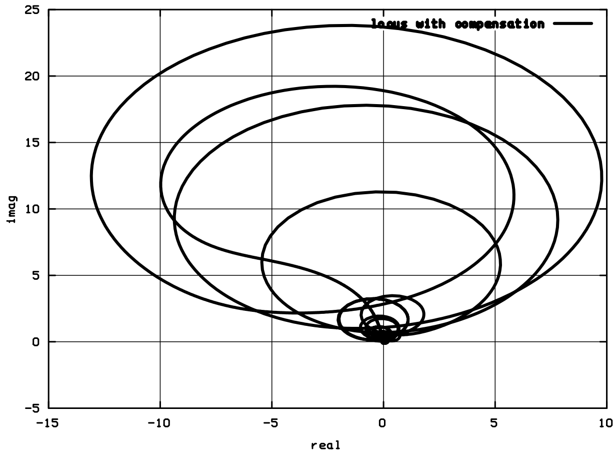

figure 9: Nyquist contour of the open loop transfer function of the controller with the Smith compensator.

In figure 10, the simulation of the active noise controller in the duct is presented. The pressure during time at the error microphone is displayed. The thin line is the sound pressure without controller active with a frequency of 180 Hz and 15 kPa amplitude. When activating the controller without Smith compensation, the controller is unstable and the simulation has stopped when the amplitude becomes larger than 1 MPa, which occurs almost directly after activating the controller. When the Smith compensator is applied, the controller using the same gain remains stable, and the sound pressure is reduced to 1.3 kPa. The Smith compensator is capable to improve stability considerably, using a simplified plant wherein only transducer dynamics are taken into account.

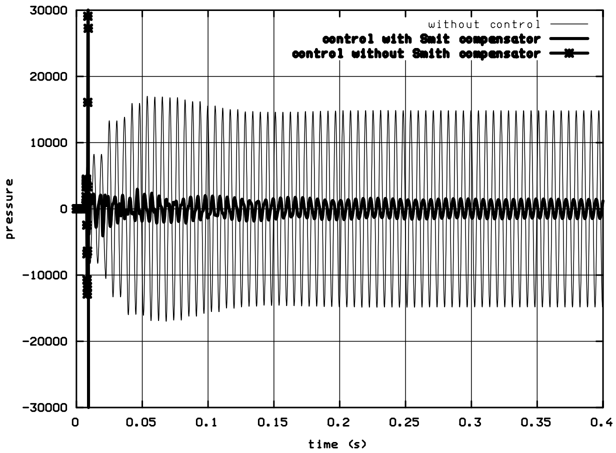

figure 10: Simulated sound pressure at the error microphone during time of a proportional active noise controller in a duct, with and without Smith compensation.

4. Experimental issues

Experiments are carried out to implement a proportional controller with a Smith compensator on the duct system. The duct system has the same dimensions as described in the simulation section. The experiments were not successful in the sense that a closed loop could not be achieved. The measurements shown in the figures are open loop measurements. The main reason that the loop could not be closed is the poor dynamic behaviour of the loudspeaker as demonstrated in figure 11. The diaphragm of the loudspeaker exhibits several modes in the working frequency range, starting from 300 Hz, causing an continuous increase of amplitude of the loudspeaker transfer function, together with a continuous phase lag. The increase of the loudspeaker transfer function amplitude amounts 10 to 15 dB. The consequence is that it becomes impossible to make a simple controller with approximately 800 Hz bandwidth. Nevertheless, the action of the Smith compensator can be demonstrated on the measured open loop transfer functions.

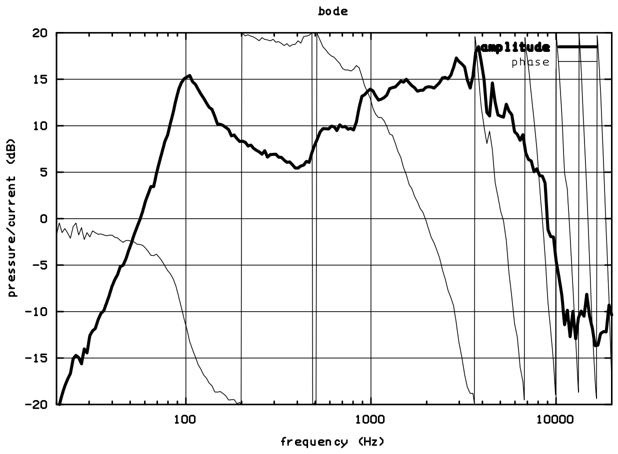

figure 11: Measured transfer function of the noise canceling loudspeaker between input coil current and output sound pressure in free space, with 0.2 m distance between microphone and loudspeaker.

The plant transfer function is presented in figure 12. The phase reaches the -180/deg at 350 Hz. When the Smith compensator is applied, the phase wrap is postponed until 800 Hz, as displayed in figure 13. If the loudspeaker dynamics could be improved, a controller with approximately 400 Hz bandwidth could be applied.

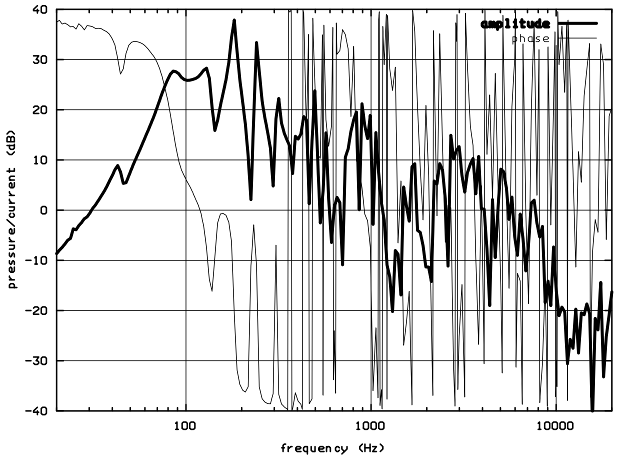

figure 12: Measured transfer function between the noise canceling loudspeaker and the error microphone in the duct.

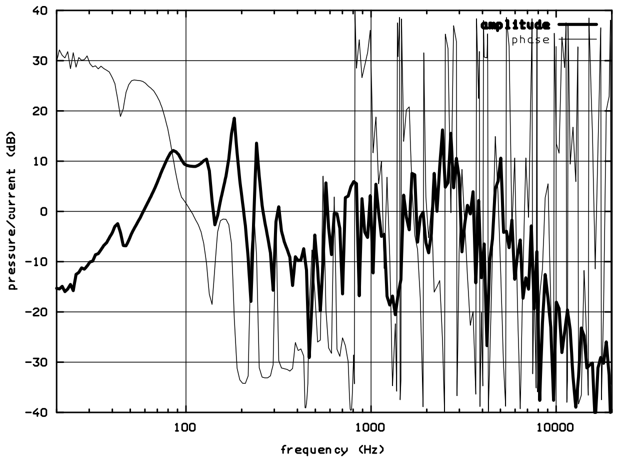

figure 13: Measured open loop transfer function between the noise canceling loudspeaker and the error microphone in the duct, with a proportional controller and Smith compensator.

5. Conclusions

This investigation leads to the following conclusions:

- The Smith compensator allows a higher control bandwidth by compensating phase lag due to time delay between error microphone and noise cancellation loudspeaker.
- The Smith compensator requires a model of the acoustic plant. A reduced plant model wherein only the transducer dynamics are taken into account is sufficient.
- The Smith compensator is connected in feedback over the controller. The closed loop formed by

the controller and the compensator must also be stable. The maximum phase lag which can be compensated is bounded by its stability criterion.

- The demands on the transducer dynamics remains high. A closed loop could not be performed in the experimental setup due to unsatisfactory loudspeaker dynamics.

References

1. A.J. Hull, C.J. Radcliffe, M. Miklavcic, C.R. MacCluer, *State space representation of the nonself-adjoint acoustic duct system*, ASME J. of Vibrations and acoustics, Vol.112,pp.483-488 (1995)
2. Zhen Wu, V.K. Varadan, K.Y. Lee, *A state space modeling of one-dimensional active noise control systems*, ASME J. of Vibrations and acoustics, Vol.117,pp.220-225 (1995)
3. R.L. Clark, K.D. Frampton, *Phase compensation for feedback control of enclosed sound fields*, J. of Sound and Vibration, Vol.195(5), pp.701-718 (1996)
4. R.L. Clark, D.G. Cole, *Active damping of enclosed sound fields through direct rate feedback control*, J. Acoustical Society of America, Vol.97, pp.1710-1716 (1995)
5. O.J.M. Smith, *Chem. Eng. Prog.*, Vol.53, pp.217 (1959)
6. L. L. Beranek, *Acoustics*, Mc Graw-Hill, (1954)
7. G.J. Thaler, R.G. Brown, *Analysis and Design of Feedback Control Systems*, McGraw-Hill, (1960)
8. S. M. Kuo and D. R. Morgan, *Active Noise Control Systems*, John Wiley & Sons, (1996)