Optimal excitation for identification of a cam set-up

B. Demeulenaere, V. Lampaert, J. Swevers, J. De Schutter
Department of Mechanical Engineering, Division PMA, K.U.Leuven, Belgium
e-mail: bram.demeulenaere@mech.kuleuven.ac.be

Abstract

This paper describes the design of optimal excitations for the identification of a cam set-up. The goal of the identification is obtaining an accurate machine model such that it can be integrated in the design of the cam that drives the set-up. The first part of the paper discusses the identification of the cam set-up dynamics for a given excitation. The identification is done in the time domain, using a maximum likelihood estimator, and explicitly takes advantage of the periodicity of the measurements. The second part describes a methodology to optimize the excitation, which can be done in two ways: (i) optimizing the drive speed of the drive motor and (ii) optimizing the cam profile. A critical part of the optimization algorithm is the calculation of the steady state response (to the applied excitation) of the cam set-up. This is done in an iterative way in the frequency domain and requires a frequency domain description of the non-linear cam-follower mechanism, which is based on an analogy with phase modulation.

1 Introduction

Cam-follower mechanisms are often used for realizing fast, periodical motions, as they occur in looms. The fierce competition in the textile industry forces the loom manufacturers to produce increasingly faster machines, which causes problems such as inaccuracy, vibrations and wear. A major cause of these problems is the fluctuation of the drive speed of the cam-follower mechanism. Generally speaking, the cam is designed for a constant drive speed, which is a reasonable assumption in slow machines, where the drive speed fluctuation is limited. In fast machines however, these fluctuations are substantial, which results in a more pronounced excitation of the machine resonances than in the case of an almost constant drive speed.

The traditional solution to this problem is a flywheel, which keeps the drive speed as constant as possible. However, a better - mechatronic - solution is to calculate the drive speed fluctuations in advance and to take them into account when designing the cam. This approach towards cam design is significantly different from the traditional one and provides a way to develop machines with fewer problems at high drive speeds. A primary requirement to implement this approach is an accurate dynamic machine model. This model can be obtained through system identification, an experimental technique which can be formally described as [4]:

The selection of a model for a system, using a limited number of measurements of the input and outputs, which may be disturbed by noise, and a priori system knowledge.

Generally, a system identification procedure consists of three steps [4]. The first step is experiment design: designing an experiment with the purpose of collecting useful data. Theoretically, this boils down to designing a *persistent* excitation of the system, which means that the inputs should be sufficiently rich such that all modes are excited and observable in the output sequence[1]. However, appropriate experiment design goes beyond satisfying the condition of persistency of excitation: another possibility is designing excitations that are optimal according to some optimization criterion. This is discussed in section 3.

The second step is the design of an identification model, which is an equation that allows the calculation of the system dynamics on the basis of the measurements. In this case, the identification model has the following form:

$$\mathbf{\Phi p} = \tau, \qquad (1)$$

in which $\mathbf{p}$ is a column vector containing the unknown parameters that describe the system dynamics. This regression equation is explained in detail in section 2.3.

[1] As a rule of thumb, an input signal should contain at least n different sinusoids in order to identify an n-th order system.

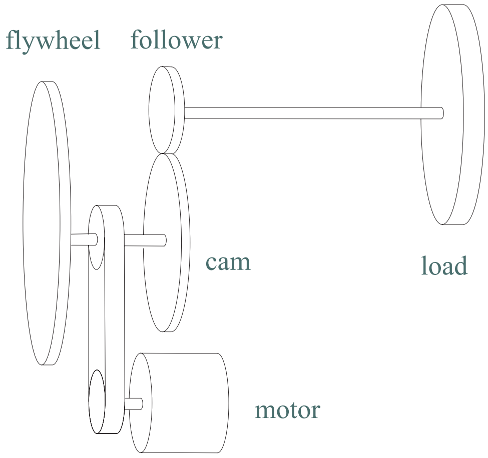

Figure 1: Physical model of the cam set-up

The third step is the estimation of the unknown parameters $\mathbf{p}$. This means calculating the vector $\mathbf{p}$ that gives the best (according to some criterion) explanation of the measured data in the regression matrix $\mathbf{\Phi}$ and the vector τ. This is explained in detail in section 2.4.

Section 2 introduces the cam set-up and describes the second and the third step of the system identification procedure sketched above. Section 3 deals with the first step of the system identification procedure: the design of optimal identification experiments (i.e. optimal excitations). The implementation of the optimization and the numerical results are the subjects of the sections 4 and 5.

2 Identification of the cam set-up

This section discusses the identification of the cam set-up dynamics. Section 2.1 introduces the cam set-up. The signal processing explicitly takes into account the periodicity of the measurements. This is explained in detail in section 2.2. Section 2.3 discusses the derivation of the identification model (the second step in the system identification procedure), and section 2.4 deals with the estimation of the unknown parameters $\mathbf{p}$ (the third step in the system identification procedure).

2.1 Description of the cam set-up

Figure 1 gives a schematic representation of the cam set-up. The set-up (a simplified model of the drive train of a loom) consists of two main parts: a drive train, and a follower train. A cam-follower mechanism connects both parts of the set-up.

The drive train consists of a DC motor that drives a camshaft via a belt (reduction ratio β). The belt is considered to be perfectly stiff. The camshaft contains a flywheel and a cam that drives an oscillating cam follower.

The follower train, consisting of a torsion rod and an oscillating load inertia, has a sharp 15 Hz resonance. By driving the cam with a speed of 180 rpm (3 Hz), the fifth cam harmonic excites this resonance ($3\text{Hz} \cdot 5 = 15\text{Hz}$). The ultimate goal of the identification is obtaining a machine model that is accurate enough to re-design the cam in such a way that the oscillating load performs some desired motion despite the 15 Hz resonance.

2.2 Handling periodical signals

Since the cam follower performs a periodic movement, all measured signals are periodic. This has some important consequences for the processing of the measurements.

First of all it is possible to improve the signal to noise ratio by averaging the measurements. Averaging a signal with variance σ^2 over n_s periods, yields an averaged signal with variance $\frac{\sigma^2}{n_s}$.

Secondly, the periodicity allows estimation of the noise properties. When averaging a signal over a large number of periods, the averaged signal has a noise level that is so low that it can serve as a reference for calculating the noise level of a single signal period.

Finally, the periodicity allows analytical calculation of the derivatives (velocities and accelerations) of the measured signals (positions), which is much more accurate than numerical differentiation.

Analytical differentiation is done as follows. Since the measured positions $\theta(t)$ are periodic with period T= $\frac{2\pi}{\Omega_b}$, they can be approximated by a finite Fourier series:

$$\theta(t) = \sum_{k=1}^{K} A_k \sin(k\Omega_b t + \phi_k). \qquad (2)$$

The amplitudes A_k and phases ϕ_k can be calculated with e.g. the `fft`-algorithm[2] in *Matlab*. When these amplitudes and phases are known, the velocity $\omega(t)$ and acceleration $\alpha(t)$ are given by:

$$\omega(t) \quad = \quad \sum_{k=1}^{K} k\Omega_b A_k \sin(k\Omega_b t + \phi_k + \frac{\pi}{2})$$

[2]Since the signal is perfectly periodical, there are no leakage errors.

$$\alpha(t) = \sum_{k=1}^{K} (k\Omega_b)^2 A_k \sin(k\Omega_b t + \phi_k + \pi)$$

Choosing a limited K in equation (2) boils down to choosing the main spectral lines of the signal. This has as a consequence that the irrelevant spectral lines, due to measurement noise, disappear from the signal. This improves the signal to noise ratio.

Figure 2 illustrates the difference between analytical and numerical differentiation. The measured signal is a periodic position signal containing five harmonics. The noise level is very low (noise variance: 0.001^2 rad^2), so that the noise is hardly visible in figure 2. The velocity and acceleration are calculated in three different ways. The first row shows the results for numerical differentiation, which are very poor. The second row shows the results for analytical differentiation with $K = 100$. The results are much better but still rather poor for the acceleration. Quasi-perfect results are obtained when differentiating analytically with $K = 10$, which is done in the third row. This figure shows that the choice of K is quite important when differentiating analytically. K should be chosen as close as possible to the number of relevant spectral lines (five in this case).

2.3 The identification model

A mathematical model of the cam set-up is derived on the basis of the physical model of figure 1. This mathematical model is needed for the derivation of an identification model. An identification model is an equation that allows the calculation of the system dynamics on the basis of the measured inputs and outputs, i.e. the position of the cam, the position of the load, the motor torque and the torque in the torsion rod. The identification model has the following form:

$$\mathbf{\Phi p} = \tau. \tag{3}$$

$\mathbf{\Phi}$ is a $N \times r$ regression matrix with N the number of measurements and r the number of unknown parameters. $\mathbf{\Phi}$ only depends on kinematic quantities, i.e. positions, velocities and accelerations. The positions are measured and the velocities and accelerations are obtained through analytical differentiation. τ is a $N \times 1$ column vector only depending on the measured torques. $\mathbf{p}$ is a $r \times 1$ column vector containing the parameters that describe the system dynamics. These parameters are inertias [kg m^2], viscous friction coefficients $\left[\frac{\text{Nm}}{\text{rad/s}}\right]$, and stiffness coefficients $\left[\frac{\text{Nm}}{\text{rad}}\right]$.

It is important to notice that the regression equation (3) is linear in the parameters to be estimated, despite the presence of a non-linearity, i.e. the cam-follower mechanism.

2.4 Estimating the parameter vector $\mathbf{p}$

$\mathbf{p}$ is calculated using a maximum likelihood estimator (MLE). The MLE requires a priori knowledge about the noise properties of the measurements. These are known thanks to the periodicity of the measurement (see section 2.2).

Since all measurements are subjected to noise, the estimated parameters $\mathbf{p}$ are stochastic variables. The MLE has two important stochastic properties [3]. First of all it is *asymptotically unbiased*, which means that the expected value of the estimate equals the 'true' value, when the number of measurements approaches infinite. Secondly it is *asymptotically efficient*, which means that the covariance matrix on the estimated parameters approaches the Cramér-Rao bound when the number of measurements approaches infinite. The Cramér-Rao bound is a theoretical lower limit for the covariance matrix on the estimated parameters.

Since $\mathbf{\Phi}$ can be considered to be noise free, the MLE reduces to a weighted least squares estimator (WLE), which allows a straightforward calculation in *Matlab*. The following arguments justify the assumption of a noise free $\mathbf{\Phi}$:

- The position measurements can be considered free of noise. This is justified by the fact that the position sensors deliver very clean signals. Moreover, since the position signals are periodic, averaging allows an even further reduction of the already small noise level.
- The velocities and accelerations are calculated analytically, which is much more accurate than calculating the velocities and accelerations numerically.

Simulation (in *Simulink*) shows that removing the flywheel yields a significant improvement of the identification. This can be explained by the fact that the removal of the flywheel allows a much bigger variation of the cam speed, which improves the excitation and hence the identification results. It should be pointed out that some of the parameters to be estimated are dependent of the presence of the flywheel. That's why the identification should be a two-step procedure: in a first step, the flywheel is removed, and all

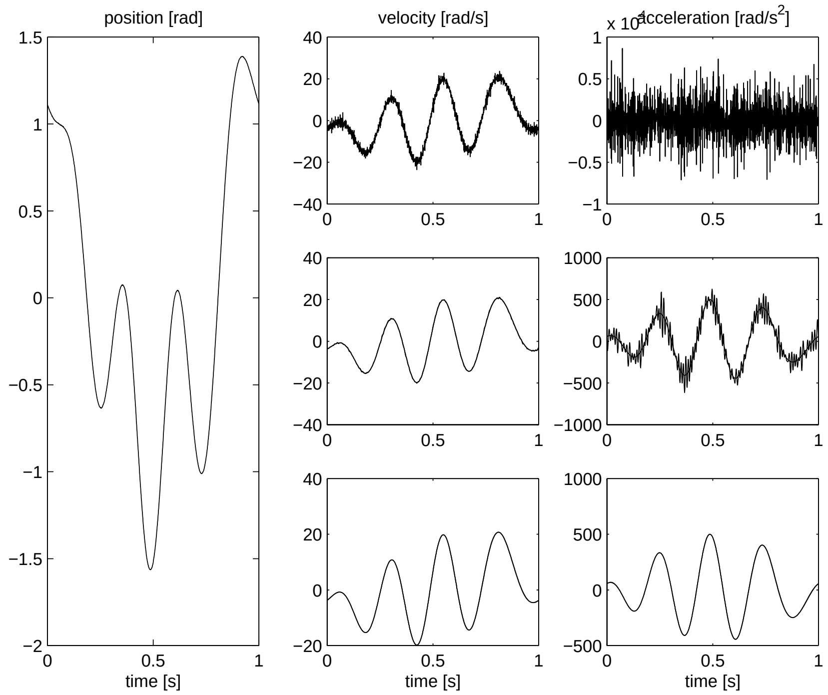

Figure 2: Comparison between numerical differentiation (first row) and analytical differentiation with a large (second row) and a small (third row) number of harmonics

parameters are estimated. In a second step, the flywheel is put back and the flywheel-dependent parameters are estimated again, with the estimated value of the flywheel-independent parameters as a priori knowledge.

3 Designing optimal excitations

3.1 Parameters to be optimized

The cam set-up is identified while being in motion. Therefore, there are two approaches to optimize the excitation [1]: (i) replacing the nominal cam with a cam with a less smooth profile, i.e. an 'identification' cam and (ii) replacing the constant desired motor speed with a varying desired motor speed.

The identification cam is entirely determined by the motion law it imposes. The motion law $\theta_2 = f(\theta_1)$ is parameterized as a finite Fourier series:

$$\theta_2 = f(\theta_1) = \sum_{l=1}^{L} D_l \sin(l\theta_1 + \psi_l), \qquad (4)$$

where θ_2 and θ_1 are the angular positions of the follower and the cam respectively. Hence, when optimizing the cam, the parameters to be optimized are the amplitudes $D_1, \ldots, D_L$ and the phases $\psi_1, \ldots, \psi_L$.

The desired motor speed ω_d is also parameterized as a finite Fourier series:

$$\omega_d = \frac{\Omega_b}{\beta} + \sum_{k=1}^{K} A_{\omega_d,k} \cos(k\Omega_b t + \phi_{\omega_d,k}), \qquad (5)$$

where β is the reduction ratio of the belt and Ω_b is the desired average speed of the camshaft. In this case, the parameters to be optimized are the amplitudes $A_{\omega_d,1}, \ldots, A_{\omega_d,K}$ and the phases $\phi_{\omega_d,1}, \ldots, \phi_{\omega_d,K}$.

3.2 Goal functions

Two different goal functions are considered: a deterministic and a stochastic one. The deterministic goal function is the condition number of the regression matrix $\mathbf{\Phi}$, which should be as small as possible since it determines to which extent relative errors on $\mathbf{\Phi}$ and τ give rise to relative errors on the estimated parameters $\mathbf{p}$ [2].

The uncertainty on the estimated parameters $\mathbf{p}$ is used as the stochastic goal function. This uncertainty is expressed as $\ln(\det(\mathbf{C})$: the logarithm of the determinant of the covariance matrix $\mathbf{C}$ on the estimated parameters $\mathbf{p}$. $\mathbf{C}$ is given by the following expression [2]:

$$\mathbf{C} = \left(\mathbf{\Phi}^T \mathbf{\Sigma}^{-1} \mathbf{\Phi}\right)^{-1}, \qquad (6)$$

where $\mathbf{\Sigma}$ is the covariance matrix of the vector τ. This stochastic goal function is the so-called *d-optimality criterion*. The d-optimality criterion has two interesting features: (i) the minimum is independent of the scaling of the parameters and (ii) $\det(\mathbf{C})$ has a physical interpretation: it can be related to the volume of the zone with the highest probability of the parameters.

3.3 Constraints

When optimizing the desired motor speed ω_d, there is only one constraint: the motor torque should be smaller than 10 Nm, since this is the maximal torque the motor can deliver.

When optimizing the identification cam, there are two constraints: the torque constraint and a constraint on the cam profile itself. This profile constraint implies that the identification cam can be made of the same casting as the nominal cam, which corresponds to a constraint on the radius of curvature: the radius of curvature of the identification cam may not differ more than 3 mm from the radius of curvature of the nominal cam.

3.4 The optimization algorithm

The optimization problem is solved using the *Matlab* `constr`-function. `constr` uses a Sequential Quadratic Programming (SQP) method. In this method, a Quadratic Programming (QP) subproblem is solved at each iteration. This algorithm does not guarantee a global optimum.

4 Implementation of the optimization

An important aspect of solving the optimization problem is the calculation of the steady state response of the cam set-up, since this is required for calculating the goal function and the constraints. The calculation of the steady state response is iterative and is performed in the frequency domain (section 4.2). This has the advantage that the steady state response is calculated in a direct manner.

The main difficulty of this frequency domain approach is imposed by the non-linearity introduced by the cam-follower mechanism. In section 4.1 a frequency domain description of the cam-follower mechanism is developed, on the basis of an analogy with phase modulation.

4.1 Frequency domain description of cam-follower mechanisms

The motion law imposed by the cam is given by equation (4). Deriving this equation with respect to the time yields $\omega_2 = f'(\theta_1)\,\omega_1$ with ω_1 and ω_2 the angular velocity of the cam and the follower respectively. The frequency domain description of the cam-follower mechanism boils down to the calculation of the frequency spectrum of ω_2, which equals the convolution of the spectra of ω_1 and $f'(\theta_1)$. The spectrum of ω_1 can easily be determined from the spectrum of θ_1:

$$\theta_1 = \Omega_b t + \sum_{k=1}^{K} A_k \cos(k\Omega_b t + \phi_k). \quad (7)$$

This equation can be understood by noticing that the flywheel is not infinitely large, so ω_1 is not perfectly constant. This implies that θ_1 is not perfectly linear but linear with some ripple (parameterized as a finite Fourier series) superimposed on in.

The main problem involved in calculating the spectrum of ω_2 is the calculation of the spectrum of $f'(\theta_1)$. Taking into account equations (4) and (7), $f'(\theta_1)$ can be written as follows:

$$\sum_{l=1}^{L} lD_l \cos\left[l\left(\Omega_b t + \sum_{k=1}^{K} A_k \cos(k\Omega_b t + \phi_k)\right) + \psi_l\right] \quad (8)$$

The calculation of the spectrum of $f'(\theta_1)$ is based on a formula normally used for the analysis of phase modulated signals [5] :

$$\exp(jz\cos\zeta) = J_0(z) + 2\sum_{n=1}^{\infty} j^n J_n(z)\cos n\zeta,$$

where j is the imaginary unit and $J_n(z)$ the Bessel function of order n evaluated in the point z. Applying this equation to equation (8) yields:

$$f'(\theta_1) = \Re\sum_{l=1}^{L}\left[lD_l\cos(l\Omega_b t + \psi_l)\prod_{k=1}^{K}\left\{J_0(lA_k) + 2\sum_{n=1}^{\infty} j^n J_n(lA_k)\cos[n(k\Omega_b t + \phi_k)]\right\}\right]. \quad (9)$$

Calculating the spectrum of $f'(\theta_1)$ boils down to multiplying out this equation. For this purpose, it is necessary to eliminate the infinite summations

$$J_0(lA_k) + 2\sum_{n=1}^{\infty} j^n J_n(lA_k)\cos[n(k\Omega_b t + \phi_k)]$$

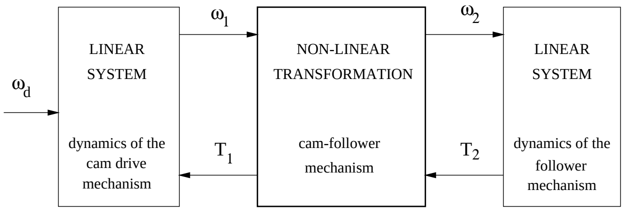

Figure 3: Iterative calculation of ω_1

in equation (9). This is done on the basis of an energy consideration: the finite summation must contain 99.99% of the energy in the infinite summation. This energy equals:

$$J_0^2(lA_k) + 2\sum_{n=1}^{\infty} J_n^2(lA_k). \qquad (10)$$

However, Bessel functions have the following interesting property:

$$J_0^2(\gamma) + 2\sum_{n=1}^{\infty} J_n^2(\gamma) = 1,$$

so that the energy of the infinite summation equals one. Replacing the infinite summation (10) by a finite summation thus boils down to selecting the smallest integer M such that

$$J_0^2(lA_k) + 2\sum_{n=1}^{M} J_n^2(lA_k) \geq 0.9999.$$

The implementation of (9) is done in *Matlab*, and takes into account the order of magnitude of the different factors, in order to reduce the number of multiplications needed for multiplying out (9). This reduces the calculation time (on a UNIX workstation) from the order of magnitude of minutes to the order of magnitude of seconds.

4.2 Iterative calculation of the steady state response

The calculation of the steady state response is done by calculating the steady state value of ω_1 in the frequency domain. All other signals can then be derived from ω_1. ω_1 is parameterized as:

$$\omega_1 = \Omega_b + \sum_{k=1}^{K} A_{\omega_1,k} \cos(k\Omega_b t + \phi_{\omega_1,k})$$

Calculating ω_1 thus comes down to calculating the amplitudes $A_{\omega_1,1}, \ldots, A_{\omega_1,K}$ and phases $\phi_{\omega_1,1}, \ldots, \phi_{\omega_1,K}$ that determine the spectrum of ω_1. Figure 3 gives a schematic overview of this iterative calculation: starting from an initial guess of ω_1 (i.e. an initial guess of its spectrum), the spectrum of ω_2 can be calculated using the approach of section 4.1. ω_2 gives rise to a torque T_2 on the follower shaft. The spectrum of this torque can be calculated on the basis of a simple linear system, i.e. the dynamics of the follower train. The torque T_1 on the camshaft can then be calculated from T_2 using the same algorithm as for the calculation of ω_2 ($T_1 = f'(\theta_1)T_2$). A simple linear system, i.e. the drive train, then transforms T_1 back into ω_1. The iteration goes on until the new estimation of ω_1 is very close to the previous estimation.

5 Optimization results

Table 1 gives an overview of the numerical optimization results obtained for the set-up shown in figure 1. It shows that optimizing the cam profile yields better results than optimizing the desired motor speed ω_d. The improvement of the goal function seems rather small, but it has to be pointed out that most probably the optimization algorithm has stopped in a local minimum. Reformulating the optimization problem in such a way that it becomes a convex optimization problem (with a guaranteed global minimum) is a subject of further research.

Another problem - besides the occurrence of local minima - is the fact that the optimization requires the values of the system parameters in order to calculate the goal function and the constraints. This has as a consequence that the optimization is an iterative process.

Figure 4 clarifies the iterative character of the optimization. Starting from an initial guess of the unknown system parameters, an optimal desired motor speed ω_d is calculated, and an experimental identification is performed. This yields improved estimates of the system parameters, and hence a new optimal ω_d can be calculated. This iterative optimization of ω_d goes on until there is no further improvement of the accuracy of the system parameters. If the system parameters are accurate enough (i.e. accurate enough for the integration of the machine dynamics into the cam design), the iteration stops. If not, an identification cam is designed, on the bases of the (improved, but not accurate enough) estimates of the system parameters. Designing an identification cam is the final step of the optimization, since this is an expensive thing to do. After the identification cam has been in-

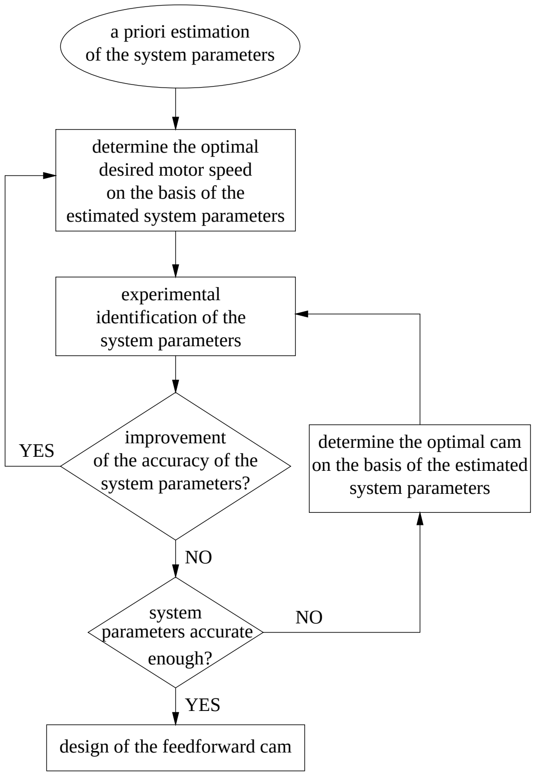

Figure 4: The iterative character of the optimization

stalled in the cam set-up, the system parameters can be estimated again and a new optimal ω_d can be iteratively designed.

6 Conclusions

Firstly a procedure has been designed to identify the system parameters of a cam set-up. This procedure is based on the equation $\mathbf{\Phi p} = \tau$ which is linear in the system parameters, despite the presence of the non-linear cam-follower mechanism. This equation remains linear, even when other non-linearities such as imbalances, motor saturation and Coulomb friction would be added to the model. The signal processing explicitly takes advantage of the periodicity of the measured signals.

Secondly, a methodology has been designed for the calculation of optimal excitations. The goal function for the optimization is either deterministic or stochastic. The calculation of the goal function and the constraint requires the calculation of the steady state response of the cam set-up. This is done in an iterative way in the frequency domain. For this purpose, a frequency domain description of cam-follower mechanisms has been developed.

Although this methodology has been developed

optimizing ω_d		
goal function	initial value	final value
deterministic	140	129
stochastic	-32	-37
optimizing the cam profile		
goal function	initial value	final value
deterministic	140	103
stochastic	-32	-39

Table 1: Numerical results of the optimization. The values in the second and the third column are respectively the initial and the final value of the goal function.

for the special case of the cam set-up considered here, it is general enough to be applicable for other kinds of machinery.

7 Acknowledgement

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