Damage localization in structures. A pattern recognition perspective.

I. Trendafilova[*] , W.Heylen, P.Sas
Department of Mechanical Engineering, Division PMA, KULeuven, Belgium
e-mail:Irina.Trendafilova@mech.kuleuven.ac.be

Abstract

The problem for damage detection and localization in structures is studied using an artificial intelligence approach. The structure is divided into sub-structures. The use of pattern recognition techniques is suggested to find the damaged substructure. The frequency response functions for a certain number of degrees of freedom for a number of frequencies are used to form the features. A mapping between the space, defined by the dynamic response of the structure in the frequency domain, and the space spanned by the features is used to develop a pattern recognition procedure. The pattern vectors and the standard samples defining the different classes are obtained using this mapping. Eventually a computer code (classifier) is built that can answer the question for the damage localization.

1. Introduction

The paper addresses the problem of developing a nondestructive procedure for damage detection in structures, based on vibration measurement data. Early damage detection and eventual estimation of damage is an important problem, since it forms the basis of any decision for structural repair and/or part replacement. The presence of even a small damage in a structure affects its dynamic behavior. Most of the quantitative global damage detection methods, that can be applied to complex structures, examine the changes in the vibration characteristics of a structure. There exists an extensive literature on the subject of damage detection using modal parameters [4,6,7] as well as frequency [1-3] and time domain responses [5,7]. Some works propose the use of the non-model-based methods for health monitoring and damage detection [8,9]. A number of papers propose the utilization of updating and identification procedures, based on a finite element model of the structure under consideration [1-3,4,7]. It is considered that the frequency response functions (FRF's) provide complete, easily assessed and adequately precise information for the dynamic behavior of a structure in a certain frequency range. There has always been a research intent to make the process of damage detection and localization as automatic as possible. Here a pattern recognition (PR) approach, that utilises the FRF response of the structure, is suggested for the purpose. It is aimed to perform the process for damage localization. Though the approach facilitates very much the process, making it nearly automatic, the final decision has to be made by the expert on the basis of the information obtained. Two different view points to the problem are considered and thus two PR problems are formulated. The structure is divided into substructures. The idea of the pattern recognition procedure is to distinguish among different categories. Two possible approaches are considered. The first one distinguishes between damaged and non-damaged substructures. The other approach differentiates among different damaged substructures. The FRF's of the structure in the pre-damaged and the post-damaged states, for a number of degrees of freedom for a number of frequencies are used to form the features. The recognition is made on a probabilistic basis, and probabilistic measures for

[*] On leave from Institute of Mechanics, Bulgarian Academy of Sciences, bl.4 Acad. G. Bontchev str.,1113 Sofia, Bulgaria

each substructure are used to determine the damaged one(s). Thus, both methods provide damage indicators between zero and one for each substructure, on the basis of which one can judge whether the substructure is damaged or not. This completes a stage for “rough” damage detection in which one or more damaged areas of the structure are found. A model updating procedure with the parameters of the elements belonging to the damaged areas only, is suggested for further localization of the damaged elements. Besides providing damage indicators for the sub-structures, the stage of the rough localization renders also a close enough initial approximation for the updating procedure, that substantially facilitates the further minimization process.

2. Substance of the method

When using the FRF's of the structure to develop a damage detection procedure, generally a selction of a limited number of frequency points and measurement points is made. The proper choice of measurement points and updating frequencies determines to a great extent the ultimate success or failure of any model-based damage detection procedure[2, 16].Such a choice opens a whole lot of questions and is not in the scope of this paper. We just mention here that we follow some of the general criteria [16] and choose a number of discrete frequencies close but different from the resonances to develop the following procedure. The choice of frequency values and their influence on the updating procedure are discussed e.g. in [2,16].

In order to check if the inspected structure is damaged, first a global characteristic based on the differences between the measured and the calculated FRF’s is suggested. Denote with $\boldsymbol{H_{ij}},\ i=1,2...n,\ j=1,2,...m$, the matrix of FRF’s for the healthy structure , where i stands for the number of frequencies and j - for the number of degrees of freedom and accordingly - with $\boldsymbol{H_{ij}}^*$- the matrix of FRF’s for the damaged structure. Form the matrix

$$h_{ij} = \sqrt{\frac{(\boldsymbol{H_{ij}} - \boldsymbol{H_{ij}^*})^2}{\max(\boldsymbol{H_{ij}}, \boldsymbol{H_{ij}^*})^2}} \qquad (1)$$

and then define the following parameter ε using its Frobenius norm $\varepsilon = \frac{\left\| h_{ij} \right\|_F}{N}$, where $N=n+m$ is the total number of h_{ij}'s. ε can be used as global damage indicator for the structure. If ε exceeds a certain threshold, i.e. $\varepsilon > \varepsilon^*$ - it is considered that the structure is damaged. If accordingly $\varepsilon < \varepsilon^*$ it is considered that the structure is not damaged.

When it is concluded that the structure is damaged, the procedure continues with the localization of the damage. The matrix h_{ij} is used for this purpose. The structure is divided into M substructures A_L, L=1,2,...M and first a pattern recognition procedure is developed in order to check for the damged substructure(s).

2.1 Pattern recognition

A pattern recognition procedure utilizes a mapping between the physical data space and the feature space [8,14,15]. In our case the physical data space is the space defined by the measured FRF's for the healthy structure $\boldsymbol{H_{ij}}$ and for the damaged structure $\boldsymbol{H_{ij}}^*$. The data from the physical space is transformed into features, that define the feature space. A certain number of categories is introduced for a PR procedure[10,11,12]. Then a decision algorithm is developed that in the feature space distinguishes among the defined categories.

2.2. Pattern recognition method 1.

One viewpoint to the damage location problem is to try to distinguish between damaged and non-damaged substructures. For each substructure A_L only those $h_{ij}^L, i=1,2,..,n,\ j=1,2,...,m_L$ are taken that are measured in points belonging to the substructure. Thus for each substructure A_L a number of variables h_{ij}^L is obtained that characterize the substructure and from these the feature vectors are to be created. The first two moments of each set h_{ij}^L are used to form the feature vectors $\boldsymbol{f}^L$. The vectors $\boldsymbol{f}^L$ for all the sub-structures are defined as follows:

$$\mathbf{f}^L = [\, f_1^L,\, f_2^L \,]'$$

$$f_1^L = M(h_{ij}^L) = \frac{\sum_{i,j} h_{ij}^L}{N^L} \qquad (2)$$

$$f_2^L = \sigma(h_{ij}^L) = \sqrt{\frac{\sum_{i,j}\left[h_{ij}^L - M(h_{ij}^L)\right]^2}{N^L}}$$

where $N^L = n.m_L$ is the number of FRF relative differences h_{ij}^L, corresponding to the substructure A^L. In this way the FRF's of the pre-damaged state $\boldsymbol{H_{ij}}$ and the FRF's of the post-damaged state are mapped into a feature vector $\boldsymbol{f}^L$. The idea of introducing such features is that for a damaged structure both, the mean values and the variances of the differences between the FRF's will not be close to zero, while for a non-damaged structure - both values are expected to be close to zero.

The following two categories that define the classes for the PR procedure are introduced :

C_D- the class of damaged substructures

C_N- the class of non-damaged substructures.

The aim of the pattern recognition procedure is to build an algorithm, and eventually a computer code, that is able to distinguish between damaged and non-damaged structures. The computer code is the classifier for the PR procedure. A "learning" procedure is organized for the purpose, which utilizes the mapping between the response space, defined by the FRF's and the feature space f_1^L, f_2^L.

A number of vectors $\boldsymbol{f_i^d}$ representing damaged substructures is obtained, by introducing damages in substructures and calculating the corresponding FRF's $\boldsymbol{H_{ij}}^*$. Then $\boldsymbol{H_{ij}}^*$ and $\boldsymbol{H_{ij}}$ are used to obtain the corresponding feature vectors $\boldsymbol{f_i^d}$. A number of vectors $\boldsymbol{f_i^n}$ corresponding to non-damaged structures is obtained by adding 5% white noise to the responses in the time domain of the pre-damaged structure and thus obtaining the corresponding $\boldsymbol{H_{ij}}^*$. These $\boldsymbol{H_{ij}}^*$ and the $\boldsymbol{H_{ij}}$ are mapped into $\boldsymbol{f_i^n}$. $\boldsymbol{f_i^n}$ and $\boldsymbol{f_i^d}$ are used as standard samples [11,12,14] for developing a statistical classifier. The statistical classifier is a computer code, which, when presented a feature vector, determines the probabilities for the corresponding substructure to belong to C_D and C_N, i.e. to be damaged or non-damaged [11,12,13,14]. The probability that a substructure is damaged is used as its damage indicator. Thus for each substructure a damage indicator

$$k_L = \Pr ob(A_L \in C_D),\ 0 \le k_L \le 1 \qquad (3)$$

is found. A high damage indicator k_L means that it is very likely that the corresponding substructure A_L is damaged and accordingly a low damage indicator means, that there is a small chance that the substructure is damaged. Usually the substructures with k_L>0.5 should be considered as suspicious. The whole picture of damage detectors for all the substructures should be taken into account. If there are several substructures with close indicators it is likely that all of them could be damaged. Eventually the judgment which sub-structures will be considered damaged is left to the expert.

2.3. Pattern recognition method 2.

Another possible viewpoint to the problem for determining the damaged substructures is to try to differentiate among different damaged substructures. In order to develop a PR approach that utilizes this idea, the following categories, that serve as classes for the problem, are introduced:

C_1- the class of structures containing damage in A_1

C_2- the class of structures containing damage in A_2

....

C_M- the class of structures containing damage in A_M

In order to recognize among the different above defined classes a pattern recognition approach uses a mapping between the physical data space, defined by $\boldsymbol{H_{ij}}^*$ and $\boldsymbol{H_{ij}}$ and the space of the feature vectors. To obtain the feature vectors the variables h_{ij} (see eq (1)) are used. Instead of using all the quantities h_{ij} **,** which is impractical, we try to find some characteristics that will help us to distinguish among the different classes. The vector $\boldsymbol{F}$ is formed as follows:

$$\boldsymbol{F} = [F_1, F_2, ..., F_M]^T =$$

$$= \left[\sum_{j=1}^{m_1} \sum_{i=1}^{n} h_{ij}, \sum_{j=1}^{m_2} \sum_{i=1}^{n} h_{ij}, ..., \sum_{j=1}^{m_M} \sum_{i=1}^{n} h_{ij} \right]^T \quad (4)$$

where m_k is the number of DOF's corresponding to area A_k. Thus a vector $\boldsymbol{F}$ is formed that has M coordinates, where M is the number of substructures, into which the whole structure is divided. Each coordinate characterizes the difference between the FRF's for a substructure. It is expected that if a substructure is damaged the corresponding coordinate will be considerably greater than all the rest ones. The vector $\boldsymbol{F}$ is used as a feature vector for this method. In order to form the standard samples, needed in the pattern recognition procedures, a number of damages for each substructure are introduced. The corresponding FRF's $\boldsymbol{H_{ij}}^{*L}$, where 'L' stands for the substructure, are calculated. $\boldsymbol{H_{ij}}^{*L}$ and $\boldsymbol{H}_{ij}^{L}$, where $\boldsymbol{H}_{ij}^{L}$ are the FRF's for the pre-damaged state, are mapped into a number of vectors $\boldsymbol{F_i^L}, i = 1,2,..N_L$. The vectors $\boldsymbol{F_i^L}$ form the standard samples for the M classes C_i, i=1,...,M. Once the standard samples are formed they are used to obtain the discriminant functions for the classes. A probabilistic approach is used for the purpose [12,13]. Thus the discriminant functions have the meaning of probability densities. With the estimation of the discriminant functions the classifier is built. When a new feature vector $\boldsymbol{F^*}$ is presented, its values are substituted in the discriminant functions $d_i(\boldsymbol{F}^*)$. The values of $d_i(\boldsymbol{F}^*)$ are used to obtain the damage indicators p_L, L=1,2,...M, that characterize the belonging of the structure to any of the defined classes, and have the meaning of probabilities. p_L characterize the probabilities that the structure belongs to any of the defined classes, and consequently they characterize the probabilities for each substructure to be damaged. Thus the closer p_L is to 1 the more likely it is that the substructure A_L is damaged. On the contrary, if p_L is close to 0, then the smaller is the chance that A_L is damaged. The vector $\boldsymbol{F^*}$ presents a structure (by its vibration response). Hence the indicators p_L, L=1,...M, characterize the structure and they contain information for any of the substructures that shows the likeliness that the corresponding substructure is damaged. On the basis of these indicators one is supposed to take a decision which substructure(s) to regard as damaged.

Above two possible artificial intelligence approaches that employ statistical pattern recognition approaches were presented for localization of a damage in a structure using its vibration response. These procedures find the damaged substructure(s). Usually these procedures can serve for approximate damage location. The procedures can be repeated if desired dividing the damaged substructures still further. In case of damage that is distributed over a part of the structure (like crack accumulation in brittle materials) the identification of the damaged substructure(s) usually provides enough information. For the case of a concentrated damage it could be desirable to make a further localization and find the damaged element(s). For such cases a subsequent updating (identification) procedure can be applied with the elements of the damaged substructures only. Such procedure can be used for localization as well as for quantification purposes. An objective function of the following type is chosen for the purpose:

$$O(S_k) = \sum_{i,j} \Phi[F_1(H_{ij}(S_k)) - F_2(H_{ij}^*)] \quad (5)$$

where S_k are the stiffnesses of the elements of the damaged area(s), k=1,2,...n_s, and n_s<N, N being the total number of finite elements of the structure. Different choices can be made for the functions Φ, $\boldsymbol{F}_1$, and $\boldsymbol{F}_2$. In the example below the next objective function was used:

$$O(S_k) = \sum_{i,j} [\log(H_{ij}) - \log(H_{ij}^*)]^3 . \quad (6)$$

3. Case study

In order to demonstrate the way the proposed procedure works, as well as to validate the results it gives, the following example was considered. Consider a rectangular plate structure, consisting of 50 finite elements. A damage is simulated in this structure by a 90% reduction of the FE stiffness of element 18 (see Fig. 1).

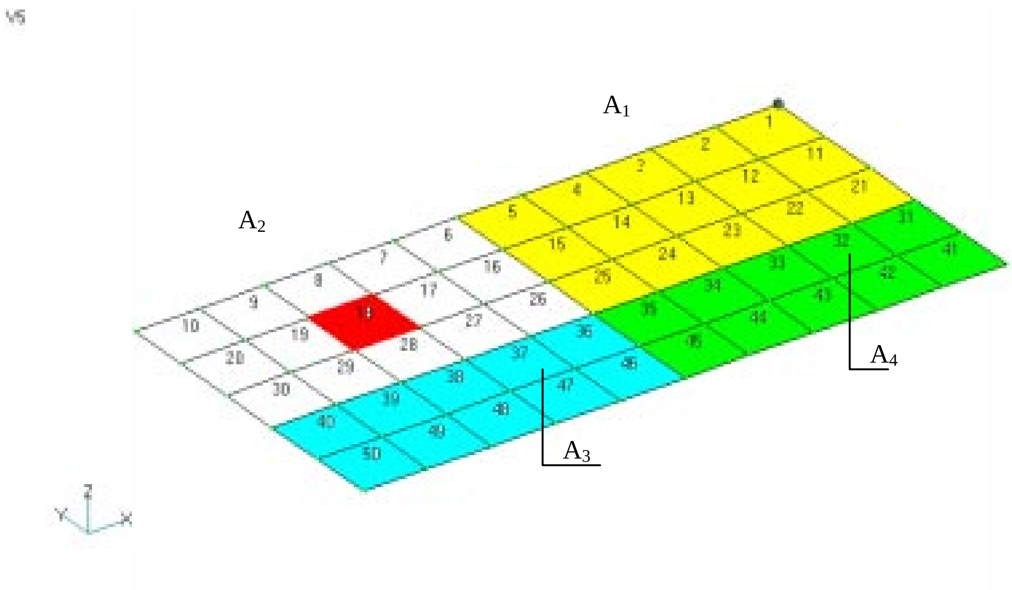

Figure 1. Case study-damaged plate, regions

The responses in FRF's for the damaged structure H^*_{ij} and the initial structure (before any damage was introduced) H_{ij} are computed. The plate is divided into four regions - A_1,A_2,A_3,A_4- A_1 and A_2 containing 15 elements each, and A_3, A_4 containing 10 elements each (see Fig.1). The first thing to do is to check if the structure is damaged. This is done as explained.above employing the global damage indicator ε. It is a number that is supposed to be between 0 and 1. For this case it is obtained $\varepsilon = 0.488$ which indicates a substantial difference between the response of the initial and the damaged structure. Thus we suppose the structure is damaged. The next step is to find out which of the areas is (are) damaged. The first method is applied and accordingly a classifier is trained to distinguish between damaged and non-damaged substructures. The discriminant functions $d_i(\boldsymbol{f})$ are obtained in the form:

$$d_j(\boldsymbol{f}) = d_j(f_1, f_2) = a_{j1} f_1 + a_{j2} f_2 + a_3 \quad (7)$$

where j=1,2. The function d_1 corresponds to the class C_D of damaged substructures and d_2- to the calss of non-damaged substructures. Convergence was reached after introducing 25 vectors $\boldsymbol{f}$ totally. Now the feature vectors $\boldsymbol{f_i}$*, i=1,2,3,4, corresponding to the areas A_i of the damaged structure (Fig. 1) are obtained and they are presented to the classifier. In order to determine the damaged areas, the classifier takes the following actions :

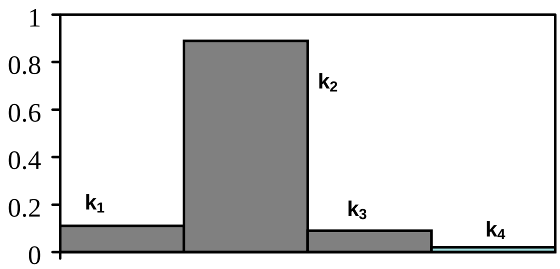

Figure 2. Damage indicators k_i for the areas A_i, i=1,2,3,4.

1. The vectors $\boldsymbol{f_i}$* are substituted in (7) and the discriminant functions $d_1(\boldsymbol{f}_i^*)$ and $d_2(\boldsymbol{f}_i^*)$, corresponding to the classes C_D and C_N, respectively, are obtained.

- ***2.*** On the basis of the discriminant functions, $d_{1,2}(\boldsymbol{f}_i^*)$ which have the meaning of probability densities, the following probabilities are obtained: • $P_j^D = Prob(\boldsymbol{f}_i^* \in C_1)$ - the probability that $\boldsymbol{f_j}$ belongs to the class C_D of damaged substructures, *and*

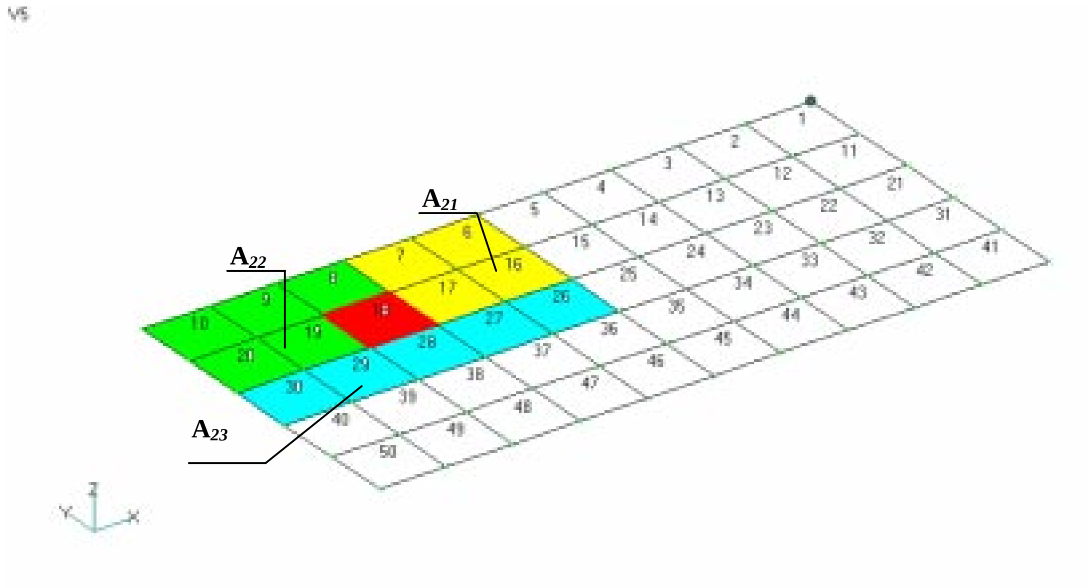

Figure 3.Sub-areas into which the damaged area A_2 is divided

• $P_i^N = Prob(\boldsymbol{f}_i^* \in C_2)$ -the probability that $\boldsymbol{f}_i$* belongs to the class C_N of non-damaged substructures. P_i^D is the probability that the corresponding area $\mathbf{A_i}$ is damaged. P_i^D are used as damage indicators for the corresponding procedure.

3. The classifier produces as results the damage indicators for all the tested areas -A_1, A_2, A_3, and A_4.

Figure 2 presents the results. It is obvious that according to these results only A_2 can be suspected as damaged. The damage indicator for A_2 (the probability that A_2 contains damage) is 0.89, which means 89 % probability that A_2 is damaged, while for all the rest of the areas these probabilities are very low (less than 10%). Accordingly it is concluded that A_2 is damaged with probability $P(A_2)=0.89$ and all the rest of the substructures are not likely to be damaged. The damage can be localized still further either by subdividing A_2 into sub-areas or by applying an updating procedure with the stiffnesses of the elements of A_2. Figure 3 gives the sub-areas into which A_2 is divided. The corresponding feature vectors $\boldsymbol{f}_{2j}$, j=1,2,3 are presented to the already built classifier. Figure 4 presents the damage indicators for the areas $\mathbf{A}_{2j}$,
j=1,2,3. The probability that the area $\mathbf{A}_{22}$ is damaged is higher, than all the rest. But the probability that $\mathbf{A}_{21}$ is damaged is also not negligible, as it was with the areas A_i. This is because the damaged element in this case is on the boundary between $\mathbf{A}_{21}$ and A_{22}. Thus, on the basis of the information that the classifier gives, in this case it cannot be concluded that just $\mathbf{A}_{22}$ is damaged and all the rest sub-areas are non-damaged **-** $\mathbf{A}_{21}$ can also be suspected as damaged. So, in this case, the procedure of sub-division of A_2 has not lead us much further. From the results one can suppose that $\mathbf{A}_{21}$ and $\mathbf{A}_{22}$ are damaged and assume that $\mathbf{A}_{23}$ is not damaged. Thus the consequent updating procedure was performed with the elements of $\mathbf{A}_{21}$ and $\mathbf{A}_{22}$.

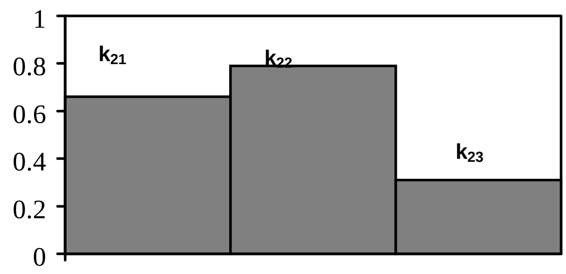

Figure 4. Damage indicators k_{2j} for the areas A_{2j}, j=1,2,3.

The procedure of further subdivision is worth performing if its results are expected to provide enough precision for the localization process, i.e. if the information that one of the substructures, obtained after the sub-division, is damaged is adequate for the purposes of the investigation. But if an elemental identification is needed, the updating procedure can be started right with the elements of A_2. It is mentioned here that the preliminary location performed by applying a PR approach makes the consequent stage of elemental updating possible by 1) restricting the damaged elements to the elements from the damaged substructure 2) by providing a close enough initial approximation for the updating. This simplifies considerably the updating procedure and makes possible its performance with a quite complicated non linear objective function (see eq.(6)).

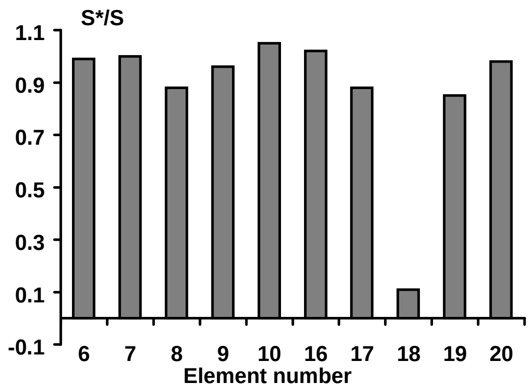

Figure 5. Updated stiffnesses for the elements of regions A_{21} and A_{22}

Since it is not the scope of this paper, the results of the updating procedure are just given below without going into details. Convergence was reached after 4 iterations. Figure 5 presents the results of the updating procedure.

As a result, the whole procedure of rough localization employing a PR approach, together with the consequent updating has provided precise enough information for the location of the damage in the structure(see Figs.1,2 and 5).

The same case study is used in order to apply the second PR procedure which tries to distinguish among different damaged areas. A classifier is trained introducing 4 classes, corresponding to the four regions into which the plate is divided (Fig.1). The discriminant functions are obtained in the form:

$$d_i(F) = \sum_j a_{ij} F_j^2 + \sum_j b_{ij} F_j + c_i$$

and convergence is reached after 15 iterations, which is considerably less compared to the number of training vectors used to build the discriminant functions for the first method. Then the vector ***F**** corresponding to the damaged structure is computed according to formula (4) and is presented to the classifier. The classifier produces the damage indicators for the regions A_i, which in this case have the meaning of the following probabilities : $p_i = \Pr ob(\textbf{the structure} \in \mathbf{C}_i),\ i = 1,2,3,4$, where C_i is the class of structures containing damage in area A_i. Figure 6 presents the results. In this case this method also succeeds in identifying the damaged area as the one with the highest damage indicator. But if one wants to sub-divide the area found as damaged, as was demonstrated above, in case this classification method is used, a new classifier with new classes is to be introduced and developed. This is one of the disadvantages of developing such a procedure. Another disadvantage of this procedure is that it supposes that only one of the substructures into which the structure is divided could be damaged.

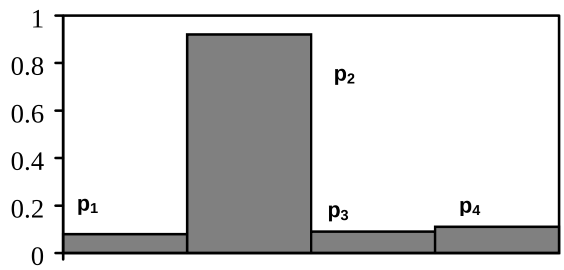

Figure 6. Damage indicators for A_i - second approach

Thus it is expected that in case of a distributed damage (like crack accumulation in some materials) or in case of more than one concentrated damage, the method could give ambiguous results which will not present valuable information for a further location and identification. But in the case of concentrated damage this procedure is expected to give more reliable results and to work faster.

4. Summary and conclusions.

The objective of this paper is to demonstrate two possible pattern recognition formulations of the problem for damage localization. The first method distinguishes between damaged and non-damaged substructures, while the other one differentiates among different damaged substructures. First the idea of a PR approach and its application to the problem for damage localization is presented. Then the substance of the two methods is presented. Both methods are illustrated on a case study of a plate with a concentrated damage simulated by a stiffness decrease in one of the elements. For this particular case both methods are able to locate the damage correctly.

The first approach has the advantage that it can be applied in cases of distributed damage, as well as in cases of multiple concentrated damages. From such viewpoint it presents a more general and practically relevant way for handling the damage detection problem. The second method presents another possible viewpoint to the damage detection problem. It has the advantage that it does not put any restrictions for the number of substructures into which the structure is divided and on their size, since it uses the same measurement points to form the feature vectors for the substructures. While for the first method any considered substructure has to be large enough and have adequate number of points susceptible to measurements in order to obtain enough information. Accordingly the second PR procedure is expected to converge faster and give more reliable and precise results for most practical situations of concentrated damage, since the pattern vectors it uses are supposed to be more informative. The second approach assumes that only one substructure can be damaged and accordingly it is not proper for cases of distributed damage or multiple defects.

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