Fault localization in structures from remote FRF measurements. Influence of the measurement points.

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Abstract

The paper considers the problem for damage detection and localization in cases when the vibration measurements are taken in points remote from the existing fault. The frequency response functions (FRF's) are used to detect and localize the damage. A localization procedure consistent of two stages - restriction of the fault in a substructure followed by a more precise elemental location - is used. The performance of the procedure and the precision it gives are analyzed and compared on a case study with simulated measurements for three sets of measurement points on the sides of the plate.

1. Introduction

Fault detection is an important problem from theoretical as well as from practical viewpoint. Fault detection methods that use the vibration response of a structure are based on the fact that even small changes in the configuration or the geometry affect the dynamic behaviour of the structure. The practical application of such methods and their employment still leaves a lot of open questions. One of them is related to the use of modal characteristics, frequency or time domain responses. A number of authors study the use of the modal characteristics of a structure to detect, localize and eventually quantify a damage in a structure [4,6]. The disadvantage of such methods resides in their insensitivity to small changes in the structure and in the problem for the precise identification of the structural modes and mode shapes. It is considered that the FRF's of a structure can provide relatively complete and precise data to be used for detecting and localizing a fault in a structure [2,3,5]. The use of FRF's in damage detection procedures still leaves a number of questions to answer and problems to attend. Part of these are related to the measurement points in which the FRF's should be taken - their number, location and distance to the damage. In a number of practical situations measurements can be made in a restricted area of the structure, because some parts are either inaccessible or difficult to perform measurements at. In this paper we study the problem for damage localization from FRF's measured remotely from the damage, considering the application of a model based fault detection procedure on a case study with different sets of measurement points. First the procedure for damage localization is presented. It consists of two parts -1) restricting the damage in a substructure employing a pattern recognition (PR) procedure, 2) a consequent more precise finite element (FE) localization applying a model updating procedure. Then a case study of a quadrangle plate is introduced. WE use the FE model of the plate to obtain its vibration response. A stiffness reduction of one of the elements of the plate introduces the damage. Three sets of measurement points containing nodes on the sides of the plate are considered. The three sets contain points that have different distances from the damage. The performance of the procedure is checked and discussed for the three measurement point sets from viewpoint of its application as well as from viewpoint of the precision of the results.

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2. Damage localization procedure.

The procedure for damage localization used here consist of a PR procedure [10] to find the damaged substructure and a consequent identification (updating) process [2,3] performed iteratively to find the damaged elements. The structure is divided into substructures A_L, L=1,2..,M. It is assumed that each substructure can be either ***damaged*** or ***non-damaged***. In order to distinguish between these two categories of substructures a pattern recognition method with the above classes of substructures is developed [8,10]. The following quantities representing the differences in the FRF's between the damaged and the intact structure, for a number of frequencies and a number of DOF's, are used to form the feature vectors:

$$h_{ij} = \sqrt{\frac{(\boldsymbol{H}_{\boldsymbol{ij}} - \boldsymbol{H}^{*}_{\boldsymbol{ij}})^2}{\max(\boldsymbol{H}_{\boldsymbol{ij}}, \boldsymbol{H}^{*}_{\boldsymbol{ij}})^2}} \tag{1}$$

where ***i*** stands for the frequencies, i=1,2,...,n, and j-for the DOF's, j=1,2,...,m

For each substructure A_L only those $h^L_{ij}, i = 1,2,..,n,\ j = 1,2,...,m_L$. are taken that are measured in points belonging to the substructure. The first two moments of each set h^L_{ij} form the feature vectors $\boldsymbol{f}^L$. The vectors $\boldsymbol{f}^L$ for all the sub-structures are defined as follows:

$$\boldsymbol{f}^L = [f_1^L, f_2^L]'$$

$$f_1^L = M(h^L_{ij}) = \frac{\sum\limits_{i,j} h^L_{ij}}{N^L} \tag{2}$$

$$f_2^L = \sigma(h^L_{ij}) = \sqrt{\frac{\sum\limits_{i,j}\left[h^L_{ij} - M(h^L_{ij})\right]^2}{N^L}}$$

where $N^L = n.m_L$ is the number of FRF relative differences h^L_{ij}, corresponding to the substructure A^L. A statistical classifier [8,9,10] is developed which gives, when presented a feature vector f^K, the probabilities that the corresponding substructure A^K is damaged - P^K_D or non-damaged- P^K_N. The PR procedure results into two quantities P^L_D, P^L_N for each substructure that characterize its state of damage. The quantity P^K_D is used as a damage indicator for the substructure A^K. The lower P^K_D the less likely it is that A^K is damaged. When P^K_D approaches 1 the chances that A^K is damaged become very high. On the basis of the damage indicators P^K_D, K=1,2,...M for all the substructures, it is decided which substructure(s) will be considered damaged. Normally all the substructures with damage indicators higher than 0.5 should be suspected as damaged. But eventually the expert should decide taking into account all the information the classifier gives and all the knowledge for the structure.

The second part of the procedure consists of an identification process with the elemental parameters [3,6,7] characterizing the damage. This allows to detect the damaged elements and can be used also to characterize the quantity(magnitude) of the damage.

3. Case study.

Consider a quadrangle plate structure represented by the FE model shown on Fig.1. A damage is introduced by a 90% stiffness reduction in element 29 (Fig.1). The example is quite simple since in this paper we aim to emphasize on the influence of the measurement points on the identification procedure. The following three sets of measurement points are introduced:

1) Set I - all the 11 nodes on AB - 11 points totally (Fig.1);
2) Set II - Set I plus the other 10 nodes on BC (the node in B was already introduced in set I)- 21 points totally (Fig.2);
3) Set III - Set II plus the other 10 nodes on CD (the one in C was already introduced in set II) - 31 points totally (Fig.5).

- Set I consists of points that are quite distant from the damage. The shortest distance to the damaged element for this set *0.7a*, where *a* is the dimension (width and length) of the plate. If the differences h_{ij} are inspected it can be observed that only for the two measurement points that are on distance *0.7a* from the damage the quantities h_{ij} are affected by the presence of the damage, while all the other FRF's are not changed.

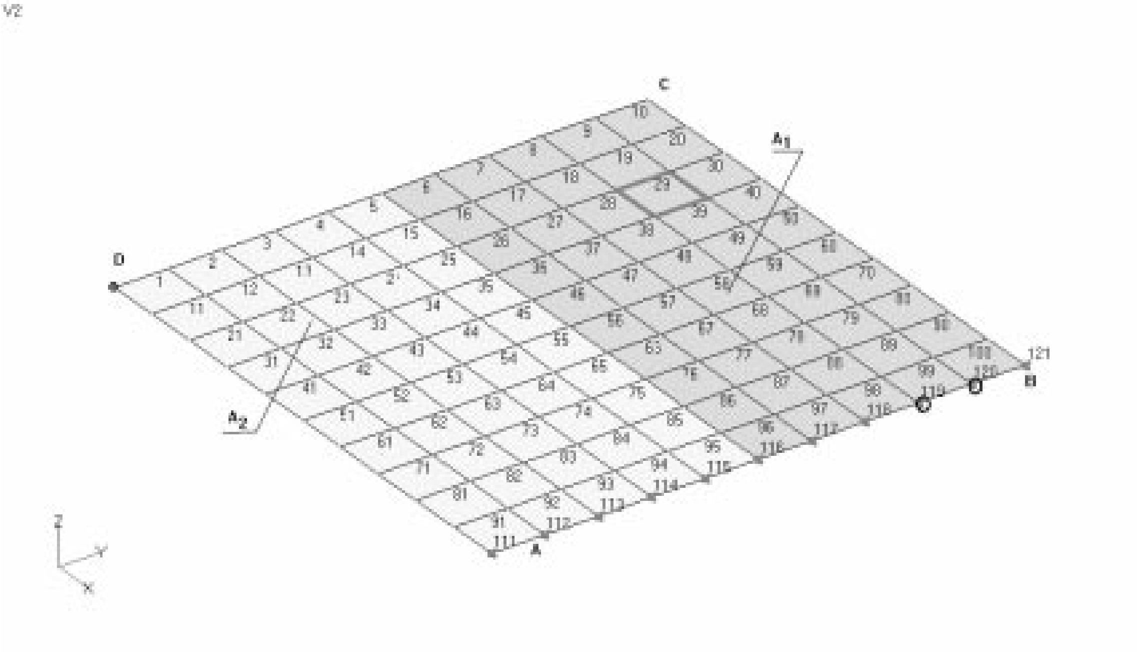

Figure 1. Case study -Set I, damaged element, areas

These measurement points coincide with nodes 119 and 120 and are shown on Fig.1 with circles. It can be observed that the distances of these two nodes to the damaged element are made of the lengths of seven (7) plate elements which makes 0.7BC=0.7a. If one looks at the affected FRF's for the different frequencies, one will observe that only the lower frequencies (up to 25 Hz, which is close to the first modal frequency) are affected by the presence of the damage. This is in accordance with some theoretically discussed results in [1,12]. As a whole the information available from this set of measurement points is scarce and is consequently expected to be inadequate for the proper localization and estimation of the present damage.

- Set II contains points that are closer to the existing damage. Considering the measurement points on BC, the only point for which the FRF's are not affected is B. For all the rest of the measurement points the FRF' are more or less affected by the presence of the damage. One can also notice, that B turns to be the only point on BC that is out of the *0.7a* distance range from the damage. For this set the frequency range of affected FRF's goes up to much higher frequencies. Accordingly it can be considered that, compared to set I, set II contains considerably more information for the FRF changes caused by the damage and thus can be expected to serve better as a localization and an identification basis.
- Set III contains the largest number of measurement points. Looking at the FRF's for the points on CD, only those for the two last nodes close to D and those for D are not affected by the introduced fault. The FRF's for a quite wide frequency range are affected by the damage. This means that set III is expected to contain the most complete information in terms of FRF's for the change in the vibration characteristics of the structure due to the damage.

3.1 Localization and estimation with set I.

The localization procedure as described in section 2 is performed with the FRF's for the points from set I. Due to the measurement points (Fig.1) the plate can be divided only along AB. Accordingly it was divided into two areas A_1 and A_2 dividing AB into two equal parts (Fig.1). The results from the first part of the localization procedure yield the following damage detectors (probabilities that the corresponding area is damaged) for the areas A_1 and A_2:

$$P_1^D = 0.31; \qquad P_2^D = 0 .$$

Thus only A_1 can be suspected as damaged.

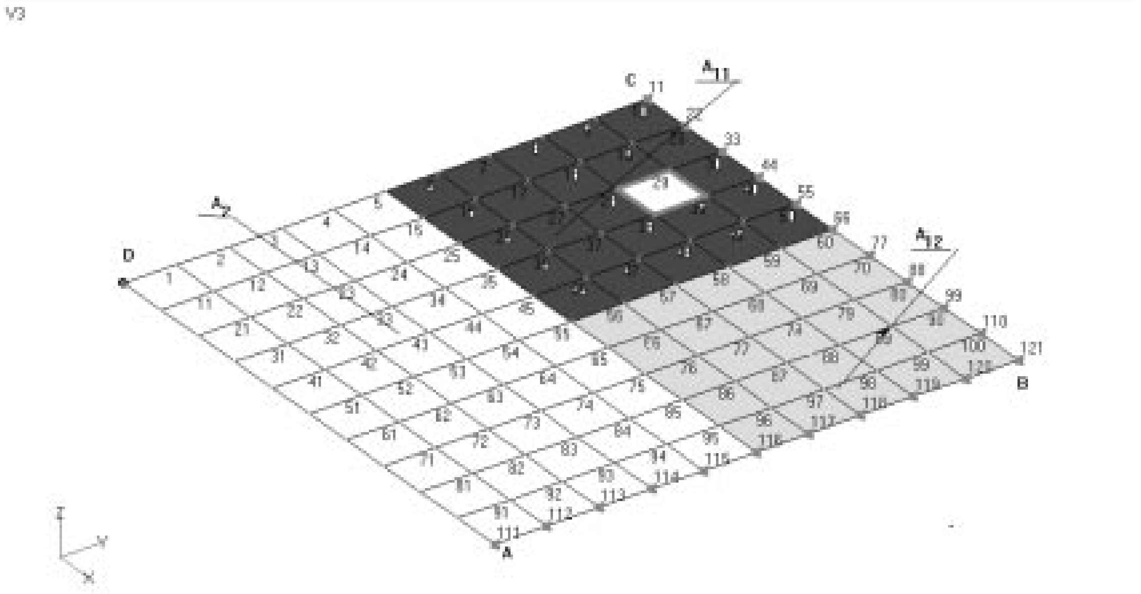

Figure 2. Measurement points from set II, areas

And this is natural since the FRF's for all the measurement points from set I belonging to A_2 are, as mentioned, unaffected by the present damage. Accordingly 0 probability that A_2 is damaged is obtained. For A_1 there are two measurement points for which the FRF's are affected slightly, all the rest are unchanged. Thus a very low probability of 31% that A_1 is damaged is obtained.

From this stage of the identification with set I measurement points there is just one conclusion that is reached: ***From the available information A_2 is intact and A_1 can be considered as damaged.*** This conclusion still leaves a lot of questions open, but for the case considered it is true.

A consequent updating procedure with the elemental stiffnesses of A_1 was attempted as a second step of the localization procedure, but it was unsuccessful. The procedure was tried with different objective functions and different iterative procedures but it failed mainly due to the insensitivity of the FRF residues (eq.(1)) to the introduced damage. Thus the whole identification procedure with the data from set I ended up with the conclusion reached on the first stage, namely that there is a 31% probability that the damage is in the area A_1 of the structure and A_2 is not damaged.

3.2 Localization and estimation with set II.

With this set of measurement points (Fig.2) it is possible to divide the plate alongside BC also. Since, compared to data set I, additional information comes from the FRF's for BC , no additional data is available for A_2. Thus A_1 is divided into two areas - A_{11} and A_{12} (Figure 2). Figure 3 presents the results from the pattern recognition procedure for damage indicators for A_{11}, A_{12} and A_2. Again 0 probability that A_2 is damaged is obtained and this is due to the fact that, as mentioned, the same data is used for A_2 as in the localization procedure with set I. The probabilities that A_{11} and A_{12} are damaged are clearly different and the picture indicates that A_{11} is the damage containing area with probability $P_{11}^{D} = 0.87$ while the rest two areas appear to be undamaged according to the data from set II.

The following minimization problem formulates a consecutive identification procedure with the elemental stiffnesses of A_{11}:

$$\min_{S_k^{11}} \left\{ \left\| h_{ij} \right\| \right\} = \min_{S_k^{11}} \left\{ \left\| H_{ij}^{11} - H_{ij}^{11*} \right\| \right\}, \quad (3)$$

where the superscript '11' refers to the substructure (A_{11}) and S_k^{11} are the stiffnesses of the elements from A_{11}.

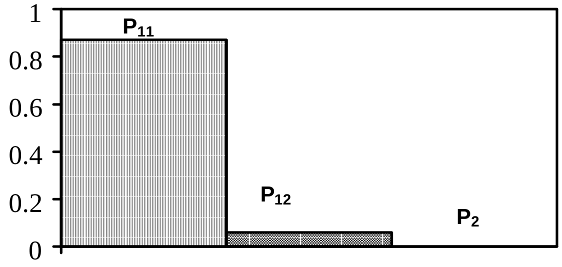

Figure 3. Damage indicators for A_{11}, A_{12} and A_2

This is a nonlinear minimization problem. Solving it, one aims at obtaining the updated stiffnesses S_k^{11*} of the elements of A_{11} for the damaged substructure. Due to the character of the FRF residues [11], the successful solution of this problem strongly depends on the initial approximation used to organize the iterative procedure. A close enough initial approximation is provided in this case by the PR procedure used to identify the damaged substructure.

Fig.4 presents the results of the updating procedure. It contains the relative stiffness changes for the elements from A_{11} as well as the relative errors between the introduced and the obtained stiffnesses. The relative stiffness changes are defined as follows:

$$\Delta S_k = \frac{S_k^{11*} - S_k^{11}}{S_k^{11}}, \qquad (4)$$

where S_k^{11*} are the updated stiffnesses and S_k^{11} are the stiffnesses of A_{11} for the intact structure.

The relative errors are calculated according to the next formula:

$$e_k = \frac{\left| S_k^{11*i} - S_k^{11*o} \right|}{S_i^i}, \qquad (5)$$

where S_k^{11*i} are the introduced stiffnesses to simulate damage in the structure and S_k^{11*o} are the stiffnesses obtained from the updating procedure.

The damaged element (29) is clearly detected, from the graph of the updated stiffnesses (Fig.4), as the one with the largest stiffness decrease. It can be observed that the results yield changes in the stiffnesses of other elements, besides the damaged one. The obtained stiffnesses of the neighboring elements seem to be also affected by the present damage. The maximum relative errors are encountered for these elements. The highest relative errors for this case are about 10%. Thus if one wants to use the results from the updating procedure as a quantitative estimate for the damage, it should be kept in mind that they could posses an error of about 10%. For the case of experimental data the measurement noise will be accumulated to these errors and this will most likely result in higher errors for the estimated stiffnesses.

Summarizing, it is noticed that with ***set II*** the overall identification procedure is considerably more successful compared to the one performed with set I. It results in

1) ***localization of the damage in area A_{11};***
2) ***localization of the damaged element ;***
3) ***estimation of the damage***.

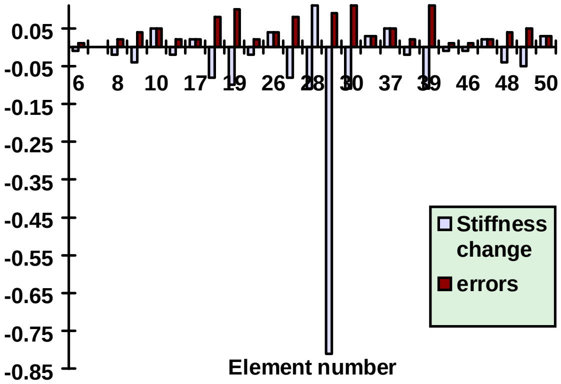

Figure 4. Stiffness changes and relative errors - set II

3.3 Localization and estimation with set III.

Set III introduces 10 more measurement points on CD (Fig. 5). Thus for the first part of the localization procedure, which is expected to restrict the damage in a substructure, the plate is divided into 4 areas (Fig.5) - A_{11}, A_{12}, A_{21}, A_{22}. The purpose of this localization procedure is to determine which of the introduced areas can be considered damaged. Fig. 6 gives the results. It presents the damage indicators - P_{11}, P_{12}, P_{21}, P_{22} - for each of the introduced areas, that give the probabilities that the corresponding area is damaged. Obviously A_{11} is detected as damaged, while all the rest of the areas show very low damage indicators. The damage indicator for A_{22} is 0, since for this area no new data is introduced, compared to

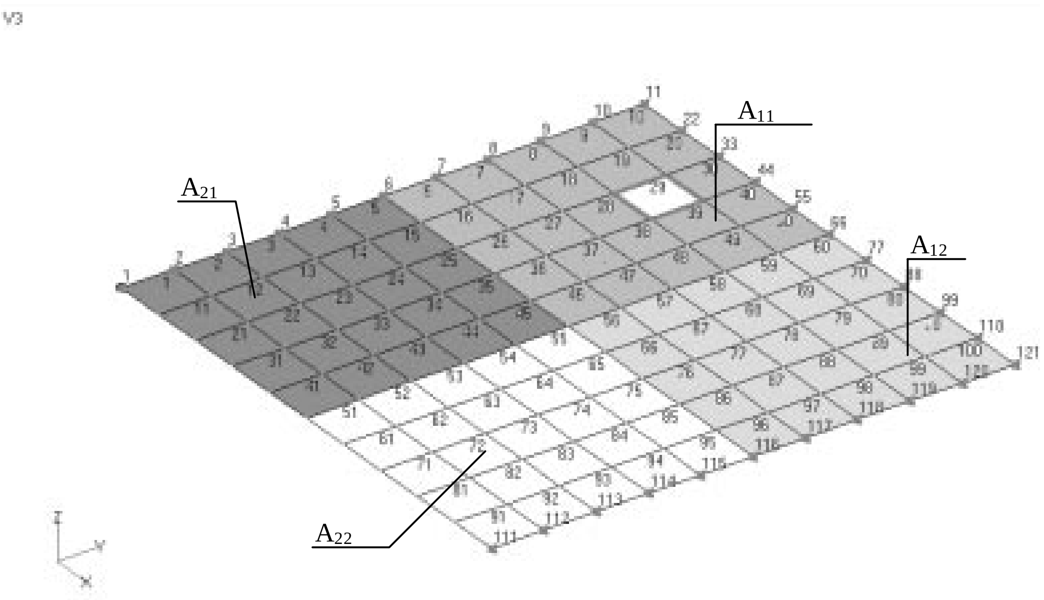

Figure 5. Set III, areas

set II, and, as was already mentioned, the FRF's in these points on AB are not affected by the introduced damage (Fig. 5). A_{11} was already identified as damaged with the FRF's from set II. So for the substructure (area) identification the bigger set of measurement points, in this case, does not give much additional information. Only for this set A_2 was divided into A_{21} and A_{22}. The damage indicators for these areas are obtained close or equal to 0, while the damage indicator for A_{11} is higher compared to the one obtained with the data from set II. Thus the results form this stage, performed with the data from set III, can give more certainty in the statement that A_{11} is damaged and the rest of the structure, covered by A_{12}, A_{21} and A_{22}, is not damaged.

The damaged area identified, the damaged element localization procedure starts. An updating procedure is organized with the same objective function as the one used with set II data (eq.(3)). It minimizes $\left\|h_{ij}\right\|$ with respect to the elemental stiffnesses of A_{11}. Figure 7 presents the relative stiffness changes (eq.(4)). It is quite clear from this graph that element 29 should be considered damaged as the one with the largest stiffness decrease. The stiffness changes experienced by the other elements are rather small and are due to errors of the updating procedure. For this case the relative errors (eq(5)) for the identification procedure are shown separately on Fig.8. It is noticed here that they have decreased considerably compared to the ones for case II. The maximum relative error has gone down to less than 5% compared to 10% for the updating procedure with data from set II.

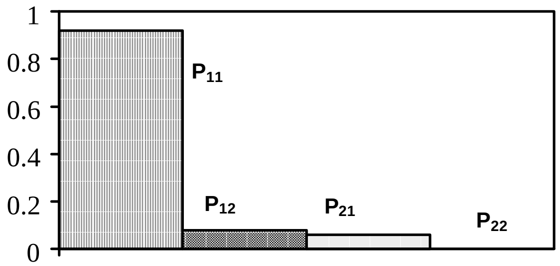

Figure 6. Damage indicators for A_{11}, A_{12}, A_{21}, A_{22}

The additional information from introducing more data from the new measurement points has contributed to the element localization and damage estimation by improving the precision of the identification process. With this set of measurement points the elemental updating can be used not only as a localization, but also as a quantification procedure for the fault. In conclusion it is summarized that the process for damage localization with the FRF's from set III succeeded in:

1) ***the rough localization detecting the damaged area;***
2) ***in the elemental localization identifying correctly the damaged element as the one with the largest stiffness decrease;***

3) ***in the quantification of the damage, identifying with high enough precision the stiffness changes, induced by the fault.***

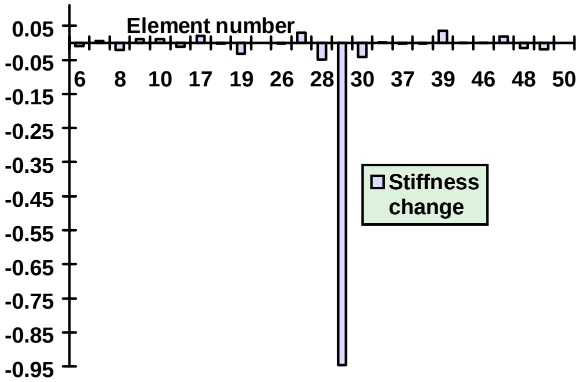

Figure 7. Relative stiffness changes- set III

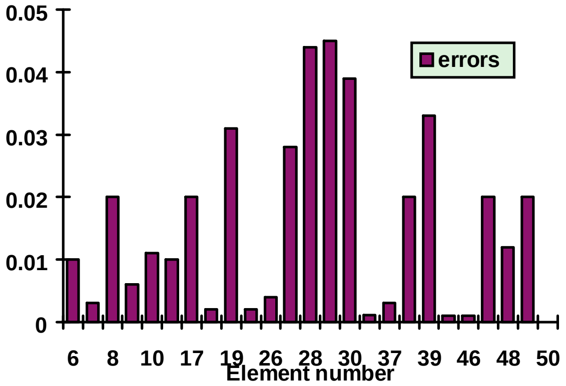

Figure 8. Relative errors - set III

Certainly for the case of experimental data one should take into account the measurement noise and keep in mind that this will most likely decrease the precision of the procedure. Nonetheless the whole identification procedure with the third set of measurement points already presents good enough precision in the localization of the damage and the estimation of its magnitude. Adding more measurement points is not expected to improve considerably the performance of the whole process, since there is not much to improve. And this brings up the idea for the choice of an optimal set of measurement points, that could give precise enough results with the least possible measurements.

4. Some comments and conclusions.

This work aims at studying the possibility for damage detection and estimation from FRF measurements in points remote from the present damage. A simple test study is considered. Three different sets of measurement points are considered. The suggested compound identification procedure constricts the defect in a certain substructure, and then localizes the damaged element. The same procedure is applied with the three sets of measurement points in order to compare its performance for differently remote locations of the points. The first measurement set consists of points quite distant from the introduced damage. There are only two measurement points in this set, for which the corresponding FRF's are affected by the damage. In this case even the procedure for constricting the damage in half of the plate hardly worked. This implies the conclusion that, as expected, if a certain distance is surpassed the damage will not influence the FRF responses taken in these points. For the example considered this threshold distance was $0.7a$, i.e. 0.7 from the dimension of the plate. Another inference is that for distant measurement points only the lower frequencies are affected, which is in accordance with some results from wave propagation theory discussed e.g. in [1,12]. Thus for set I the FRF's for frequencies lower than the first modal frequency showed a change.

Set II added more measurement points, most of which closer to the damage. For this case the identification process was considerably more successful. It provided correct substructure localization with quite high confidence of 87 %. The sequential updating procedure detected correctly the damaged element of the structure. There is more to be desired for the precision of the updating, in order to use the procedure as a quantification tool. An error of about 10 % was obtained with FE data, which is expected to increase for experimental data. This measurement set confirms that FRF's for points remote from the damage on a distance $r < 0.7a$ are affected by the damage. Here the range of affected frequencies has increased considerably, which agrees with the theoretical results considered in [1].

Measurement set III seems to provide enough information for the change in the dynamic response of the plate in terms of FRF's, so that both steps of the localization and estimation procedure were executed quite precisely. In this case the introduction of more measurement points did not contribute much to the detection of the damaged substructure, since it provided the same results as the process with set II, only with higher confidence of 93%. With this set it was possible to introduce more substructures than with sets I and II and they all proved to be undamaged with more than 90% probability. Thus the introduction of additional data in this case only increased the precision of the area restriction of the damage. Considering the consequent updating process with the elemental stiffnesses of the damaged area, it was noted that this showed a considerable improvement of the precision, dropping the relative error down to less than 5%.

In this paper the feasibility of a method for damage localization and estimation is studied for the case of remote measurements of the FRF's. The applicability of the method is tested on a simple simulated test case. As a whole the approach can ***1) identify the damaged substructure 2) identify the damaged element and eventually 3) estimate the extent of the damage by the updating of the stiffnesses of the damaged substructure***. In conclusion some important features should be highlighted:

- The inspection distance from the damage is a quite important factor for the eventual success of the localization and the estimation procedure [1]. It has to be lower than a certain threshold, which is case dependent.
- For distant measurement points only the lower frequencies are affected by the damage. Thus the higher frequencies used the shorter the inspection distance has to be.
- The precision of the proposed method depends strongly on the quality and the quantity of the information [2], which is determined by the number of the measurement points and their distance to the damage. For the case of enough measurement points close to the present damage, the updating procedure can provide precise enough results and can be successfully used as a damage estimator (set III).

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