Updating modal models from response measurements

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Abstract

A method for updating modal models from response measurements is extended from the case of a single impulsive excitation to a more general broadband excitation in the presence of secondary excitations. The original technique was based on analysis of the cepstrum of the response, as forcing function and transfer function effects are additive in the response cepstrum, and also separated if the force log spectrum is reasonably smooth and flat. Use is made of principal components analysis by singular value decomposition to separate the autospectrum of the response at each point to the dominant excitation, which is then curve fitted in the cepstral domain for its poles and zeros to give updated estimates of the FRFs. The resulting FRFs are scaled (because of including the information on zeros) and in the study gave reasonable estimates of the mode shapes when the dominant force was four times larger than the next largest.

1. Introduction

Experimental modal analysis techniques are well developed to determine the modal properties of structures such as vehicle bodies under laboratory conditions. These can then be used to update analytical models to obtain better predictions of the responses to hypothetical loadings, or in particular to predict the effects of modifications so that the latter can be optimised with minimal changes to the hardware. One application which is very sensitive to the accuracy of the model is the inverse problem of using such modal models to estimate the input forces giving rise to measured responses. Modal properties such as mode shapes and natural frequencies can change under operating conditions, so as to be different from those measured in the laboratory. An example is given by high speed ships, where the mass loading of water adjacent to the hull varies with speed through the water. In fact, most vehicles such as cars, rail vehicles etc would experience variations in joint stiffness and damping with changes in load and speed and these would give rise to variations in the modal properties.

This paper further develops cepstral methods for updating modal properties from response measurements as initially proposed in [1, 2], to make them more applicable to practical situations. Cepstral analysis gives the advantage that under certain conditions the forcing function and transfer function are additive in the cepstrum of the response, and to some extent also separated. One condition for the additivity is that the system be SIMO (single input, multiple output), while a condition which helps separation is if the logarithmic force spectrum is reasonably smooth and flat so that its cepstrum is short. The original work in Refs. [1, 2] was based on excitation of a simple beam by a single impulsive force, which did not have to be measured. In practice, however, there would always be more than one forcing function, even if one were dominant. It might in fact be necessary to ensure that a dominant force is applied. As an example, to measure the modal properties of a rail vehicle, it might be advantageous to mount a wheel with a flat, to give a periodic impulsive input which is dominant over other forcing functions. The further development is based on preliminary work done while one of the authors (RBR) was on sabbatical leave at the Katholieke Universiteit Leuven (KUL), Belgium, in 1997. In this further work a simple frame structure was excited by two broadband noise sources simultaneously in different locations, to simulate a more realistic situation, and an attempt was made to separate the responses due to these two forcing functions using singular value decomposition, so as to have a SIMO situation. The cepstral techniques could then be applied to the principal responses to extract updated modal parameters.

The proposed method, with further development, should lead to improved updating of models in developmental situations and thus reduce the need for intermediate prototypes, and expensive redesigns. It should be particularly valuable in improving the results of the inverse problem of predicting forces which give rise to measured responses in service, and thus improve the simulation of working conditions for new developments. Other methods are being investigated to achieve the same purpose (eg [3]) but the proposed method has the advantage that the predicted mode shapes are scaled, provided that the changes from the original model (measured in the laboratory or analytical) are not too great.

2. Theoretical Background

The proposed method is based on two fundamental techniques, cepstrum analysis, and principal components analysis through singular value decomposition.

2.1 Cepstrum analysis

The cepstrum is defined as the inverse Fourier transform of a logarithmic spectrum, itself the forward Fourier transform of a time signal [4].

Thus:

$$C(\tau) = \Im^{-1}\{\log(X(f))\} \qquad (1)$$

where

$$X(f) = \Im\{x(t)\} = A(f)\exp(j\phi(f)) \qquad (2)$$

so that

$$\log(W)) = \ln(A(f)) + j\phi(f) \qquad (3)$$

The abscissa τ of the cepstrum has the dimensions of time but is known as “quefrency”. If the data is sampled the Fourier transforms can be replaced by Z-transforms.

For a SIMO system, the relationship between the input and each output can be expressed in the following terms:

$$x(t) = f(t) * h(t)$$
$$X(f) = F(f) \cdot H(f)$$
$$\log(X(f)) = \log(F(f)) + \log(H(f))$$

and

$$\Im^{-1}\{\log(X(f))\} = \Im^{-1}\{\log(F(f))\} + \Im^{-1}\{\log(H(f))\} \qquad (4)$$

where x(t) is a typical output, f(t) is the input, and h(t) is the impulse response of the system for that output. Upper case variables represent the corresponding Fourier transforms, so that H(f) is the FRF of that system.

Thus the forcing function and transfer function components are additive in the cepstrum. Oppenheim and Schafer [5] have shown that the cepstrum of a transfer function may be represented in terms of the poles and zeros (in the Z-plane) as:

$$C_h(n) = -\sum_i \frac{a_i^{\,n}}{n} + \sum_i \frac{c_i^{\,n}}{n} \qquad , n > 0$$

and

$$C_h(n) = \sum_i \frac{b_i^{\,-n}}{n} - \sum_i \frac{d_i^{\,-n}}{n} \qquad , n < 0 \qquad (5)$$

where the ci and a_i are poles and zeros inside the unit circle, respectively, and the di and bi are the (reciprocals of the) poles and zeros outside the unit circle, respectively. The value at n = 0 is the log of the gain factor. The equations for another cepstral function known as the “differential cepstrum” [6] are almost the same as in Equ.(5), but without the hyperbolic l/n weighting on each term, because the function is defined as the inverse transform of the *derivative* of the logarithmic spectrum. This means that the differential cepstrum has a very similar form to the impulse response function, ie a sum of decaying complex exponentials. The differential cepstrum retains the property of the normal cepstrum of additivity of forcing and transfer functions in the response.

For minimum phase functions, there are no poles or zeros outside the unit circle, and the second of Equations (5) drops out, meaning that the cepstrum is one-sided (causal) and that the phase of the FRF is the Hilbert transform of the log amplitude and does not have to be measured separately. The cepstrum can be obtained from that of the log amplitude spectrum alone by setting negative quefrency values to zero and doubling positive quefrency values [5]. If the logarithmic force spectrum is smooth and flat its cepstrum will be short and the higher quefrency part of the response cepstrum will be dominated by the cepstrum of the FRF. In that case the poles and zeros of the FRF will be located at the same frequencies as the resonances and antiresonances in the autospectrum of the response.

In Ref.[l] it was shown that the updated poles and zeros of a modified FRF could be extracted by curve.fitting the cepstrum or differential cepstrum of the response autospectrum, and in Ref.[2] it was shown how these could be combined with an equalisation and scaling function obtained from the originally measured FRF to regenerate the updated FRF. By this means it was found possible to track changes in a number of FRFs resulting from a structural modification using response measurements only. However, this case involved excitation by a single impulsive force, and the zeros were quite visible in the response autospectra. In a more general situation, secondary forces would most likely mask deep antiresonances in the response autospectra and this is the main reason for introducing the concepts of Principal Components Analysis [7].

2.2 Principal components analysis

For responses measured in N dofs over a structure, the N x N response cross spectral matrix $\mathbf{X_{ij}}$ can be set up, with a typical element representing the cross spectrum between dof *i* and dof j , and the diagonal elements representing the autospectra. A singular value decomposition of a (square) response cross spectral matrix by eigenvalue analysis results in the relationship:

$$\mathbf{X} = \mathbf{U}\, \Sigma\, \mathbf{U'} \qquad (6)$$

Σ is a diagonal matrix of principal values representing the "power" in each of the orthogonal responses to the various excitations, and the number of significantly non-zero elements represents the number of such independent excitations. The cross spectral matrix corresponding to the kth principal value σ_k is given by:

$$[\mathbf{X}]_k = \mathbf{U}_k\, \sigma_k\, \mathbf{U}_k' \qquad (7)$$

where $\mathbf{U}_k$ is the kth column vector of U and the transpose implies complex conjugation. The diagonal elements of $[\mathbf{X}]_k$ represent the autospectra at each dof due to this principal component.

3. Measurements and Simulations

A set of measurements existed which had been made on an object representing an automotive engine subframe. They had been made for the study of Ref.[3] and involved simultaneous excitation by two random forces at points 13 and 24 out of a total of 27. Measurements of response acceleration were made at all 27 points in the z-direction (out of the plane of the basic frame). The force spectrum at point 13 was approximately 3 dB higher than that at point 24, and both were reasonably white except for some peak/notches caused by the interaction of the structure with the shakers. The measurements had been made using an LMS CADA-X system, and included simultaneous time records of all forces and responses and frequency response functions (FRFs) between each excitation point and all response points. All time records had a length of 32K (32,768) samples. The sampling frequency was 1024 Hz, so that FRFs were calculated over the frequency range 0 - 5 12 Hz (lowpass filter cutoff 400 Hz) with 2048 lines.

As the original study of Refs.[1, 2] had been made with impulsive excitation, the response autospectra were smooth, in contrast to the response autospectra using random excitation which had an additive random component determined by the number of averages, in this case having a standard deviation of approx. 1 dB corresponding to the effective number of averages of 15. Thus it was first checked that the cepstrum curve fitting algorithms could cope with this amount of noise by adding it to a typical measured FRF and fitting it with and without the noise. There was no significant difference in the curve fitted poles and zeros.

The principal components of the response cross spectral matrix were calculated in accordance with Equ.(7) and the first two were compared with the FRFs to the two driving points. It had been suspected that the first principal response might be that due to the largest force (ie that at point 13) and the second principal response that due the other at point 24, but Fig. 1 shows that this is not the case. Figure 1 compares the first two principal response autospectra at point 2 with the FRFs to points 13 and 24, respectively. This result is typical of all cases and shows that first principal response appears

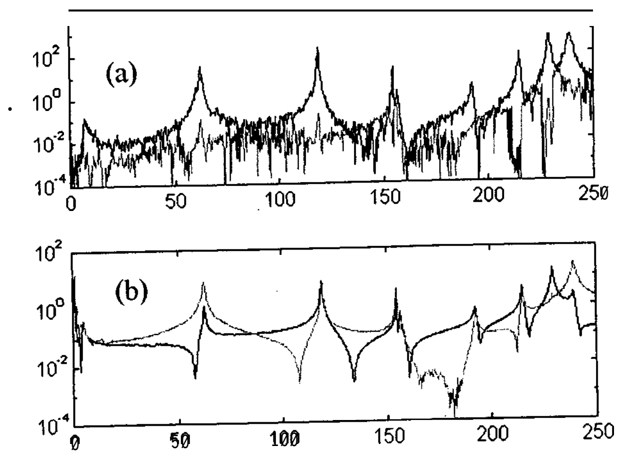

Figure 1: Comparison of first two principal response autospectra at point 2.
Upper curves - principal response autospectra (dark - first, light - second), Lower curves - FRFs (dark - from point 13, light - from point 24)

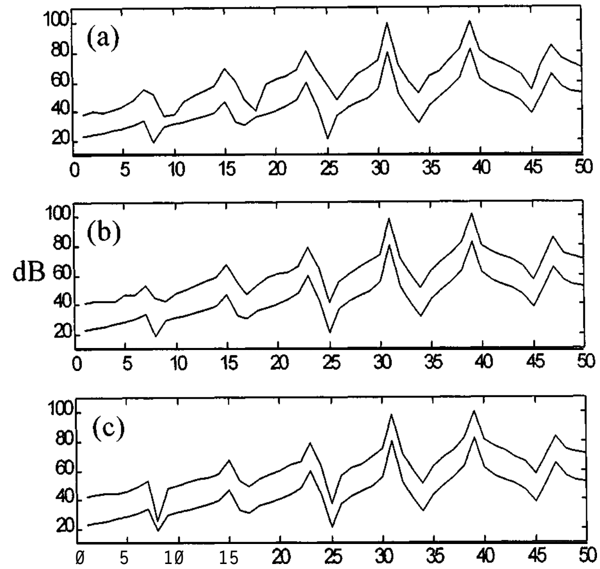

Figure 2: Comparison of first principal response autospectrum (upper) with equivalent FRF (lower), for force ratios (a) 2, (b) 4, (c) 8

to have a shape similar to one or other of the FRFs but switches between them at certain frequencies. It has the general appearance of the "envelope" of the two FRFs, but in some frequency ranges the first principal component falls below the second in the vicinity of antiresonances in the corresponding FRF. It is suspected that this is because the average response in that frequency band over all measurement points is higher for the first principal component (which it must be by definition) but the local response can be lower. Note that the second principal response does not contain much "modal" information. Thus, it was realised that for the principal components analysis to be useful, the first principal component would have to be due to a dominant force.

It was decided to carry out some simulations using a 6 dof system to determine the ratio of the highest force to the next highest in order that the former would dominate the first principal response. Figure 2 compares the first principal response autospectrum with the amplitude of the FRF at the driving point for the largest force, for force ratios of 2, 4 and 8. This was typical of the responses at all points. It can be seen that there is some deviation of zero positions for a ratio of 2, but that a ratio of 4 gives a very good correspondence, even though not quite as good as the ratio of 8. The zeros for which there is a small deviation for a ratio of 4 are at very low level as well as at low frequency.

It was thus decided to carry out a new set of measurements on the same test object, modified slightly, with the same range of force ratios of 2, 4 and 8. At the same time it was thought that different excitations in practice would be much more likely to be independent than the forces given by (relatively light) shakers fed by independent signals, but where the structural motion can give some force coupling, in particular in the vicinity of resonance frequencies. Thus a means was sought to decouple the forces supplied by the two shakers and this was found in the form of the LMS software package "Time Wave Replication" which compensates for the shaker/structure response and causes a number of specified signals to be generated. It is normally used to generate specified motions, but can equally well be used to generate specified forces. Since the method does not require fully white noise, the excitation signals were modified by a digital filter to have nominally the same spectral shape, rolling off about 8 dB from zero to the highest frequency of 400 Hz, but based on independent random signals. A small mass was added to the structure to vary its properties slightly, but comparison of the FRFs before and after indicates that the differences were somewhat greater, most likely due to a difference in the number of cemented studs of which there were a great number over the test object. In addition to recording 32K time records as before, the FRFs to both excitation points were also measured and a conventional modal analysis carried out using the CADA-X system. These form the basis of later comparisons with the results of the current study.

Figure 3 is a comparison of the two FRFs at one point (from the force at point 13) and is typical of most. It is seen that above the first two modes the variation is quite large. However, it was thought that it might be possible to use the original FRFs as a basis for determining the changes in these two modes at least, updating them by curve fitting the poles and zeros in the cepstra from the corresponding principal response autospectra. At the same time it was decided to see what reproduction accuracy could be gained over the full frequency range by using the new FRFs as a basis for determining an equalisation and scaling function (the equivalent of the "phantom zeros" mentioned above) and using the full range of poles and zeros obtained by curve fitting the principal response cepstra.

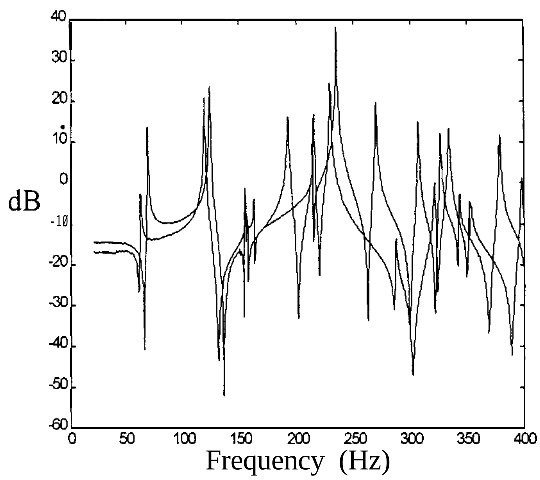

Figure 3 Comparison of FRFs for original and modified test object; excitation point 13, response point 8

Since it was necessary to use the autospectra to calculate the cepstra to be curve fitted, there was no information as to whether the zeros in the FRFs were minimum or maximum phase, although the assumption could be made that poles should be minimum phase. Two possibilities were envisaged here:

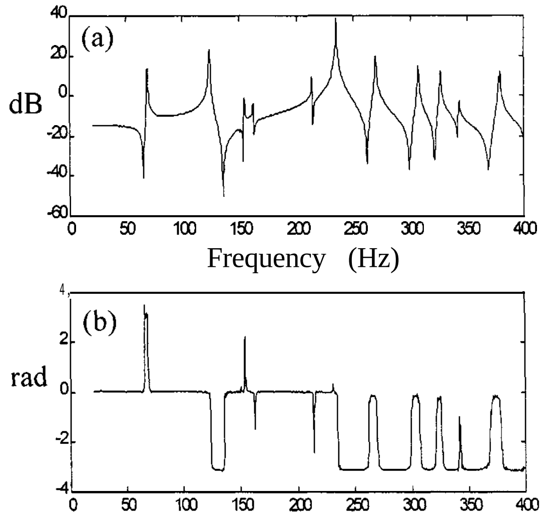

Figure 4 FRF for response point 8 after modification. (a) log amplitude (b) Phase

(a) That if all original zeros were minimum phase, the same assumption could be made for the updated model
(b) Provided poles and zeros did not shift too much, the zeros in equivalent positions could be assumed to be of the same type.

Inspection of all FRFs showed that they were minimum phase as for the typical case of that between points 13 and 8 which is shown in Fig. 4, where it is seen that the unwrapped phase changes by $+\pi$ for every zero and $-\pi$ for every pole, this being characteristic of minimum phase properties.

The first principal response autospectra were calculated for force ratio 4 in accordance with Equation 7, and these were found to correspond

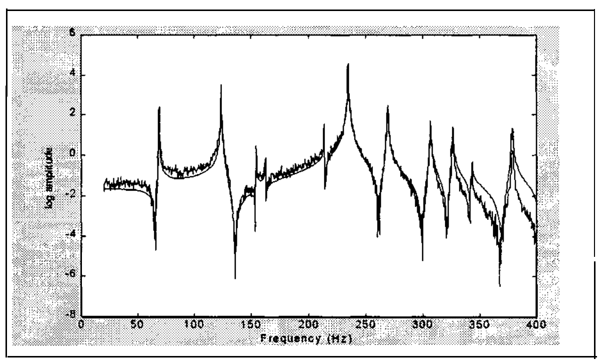

Figure 5 Comparison of first principal response autospectrum at point 8 for force ratio 4, with equivalent FRF.

very well with the amplitude of the corresponding FRFs (ie those with point 13 as excitation point) with respect to the positions of both poles and zeros. Figure 5 shows the correspondence for point 8.

4. Fitting the Principal Response Autospectra

4.1 Using Original FRFs

As seen in Fig. 3, there was very little correspondence in the FRFs before and after modification at frequencies above the first two modes, and so an attempt was made to fit only these two modes using the original FRFs. This was done by calculating the cepstra corresponding to the log amplitudes of the FRFs truncated on either side of the two modes. Such truncation was done smoothly as indicated by the regenerated FRF in Fig.6, and included any zeros immediately on either side of the two poles as in this case. The cepstrum was curve fitted for its poles and zeros (cf Equation (5)) using the non-linear least squares (NLLS) method described in Ref.[1]. The log amplitude of the FRF was then regenerated using these poles and zeros alone and compared with the original. Rather than using phantom zeros and an independent scaling factor to compensate for the difference, as in Ref.[2], a combined equalisation and scaling function was formed by smoothing the difference of the original and regenerated log amplitude spectra. Smoothing was done by lowpass filtering (Fourier transformation followed by restriction to low "frequency" values, determined empirically). Since the function being smoothed was a log spectrum, this actually corresponded to a "shortpass lifter" in the cepstrum. Next, the cepstra of the first principal response autospectra (eg see Fig. 5) were calculated and curve fitted to determine the changed values of the poles and zeros in the vicinity of the first two modes. Finally, the updated poles and zeros were combined with the equalisation and scaling functions just described to form updated FRF estimates. Figure 6 shows the result for point 8, which in fact corresponds very well. The results were not as good at a few points, in particular in the vicinity of nodes which had shifted as a result of the modification; however, the residues here would tend to be small and not greatly affect mode shapes. Furthermore, the damping was not accurately determined in some cases (eg for point 8 where the

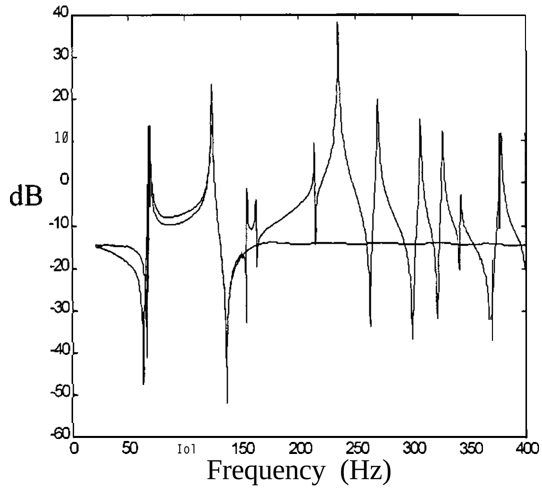

Figure 6 Regenerated FRF for first two modes at point 8, compared with measurement. Generated using original FRFs

estimate can be seen to be lower than that measured).

Even so, the complete set of regenerated FRFs were analysed for the first two mode shapes and the results are similar to those in figure 8.

4.2 Using New FRFs

To obtain an idea of how the method might function for cases where the change from the original was somewhat smaller than in this case, an attempt was made to fit the poles and zeros in the whole frequency range, but use equalisation and scaling functions determined from the new FRFs (measured after the modification). By trial and error it was found that the actual fitting of the poles and zeros was most accurate when applied to cepstra corresponding to restricted frequency ranges containing three to four modes each (separated out as described above). The equalisation and scaling function was determined for the whole frequency band, however.

Figure 7 shows the result for point 8 for the log amplitude, where the correspondence is seen to be excellent except once again for some damping estimates. In fact, some of the curve-fitted damping values came out to be maximum phase (despite being obtained from the autospectra) but were inverted before regeneration of the FRFs which were known to be minimum phase. This is possibly due to the noise in the autospectra which would also appear in the cepstra, even at high quefrency, and thus not allow the cepstra to "die out" at a rate

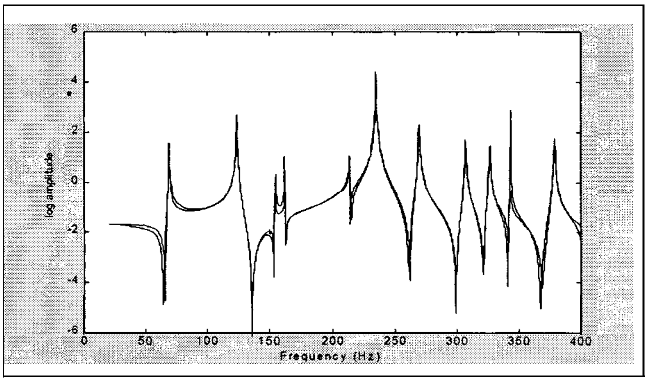

Figure 7 Regenerated FRF for point 8, compared with measurement. Generated using new FRFs

corresponding to positive damping. To test this it would be necessary to make new measurements with longer time records so as to give less random fluctuation in the autospectral estimates. Note that the noise reduction is proportional to the square root of the length of record (ie number of averages). Note that the estimated phase function has not been shown as it in all cases was virtually the same as that of the measured FRF except for minor variations corresponding to the errors in damping estimates.

Finally, the regenerated FRFs over the entire frequency band were analysed for the mode shapes, for comparison with those obtained by the LMS analysis system at the time of the measurements. Note that since the extracted FRFs are for a SIMO system, the modal analysis could be performed on the basis of a single excitation dof, even though the measurements were obtained using simultaneous excitation at two dofs. Most of the modes had a reasonable correspondence to those obtained directly, although they were somewhat more "noisy". As examples, Figure 8 compares the results for mode 2, the first bending mode, and mode 10, the second torsional mode. The obvious errors at certain points, eg point 6 for mode 2, could perhaps be corrected by smoothing the modes. Note that although it was necessary to assume an overall scaling factor for each FRF (maintained the same as for the original measurements) the fact that zeros were obtained as well as the poles means that the individual modes were scaled relative to each other; in other words the final result gives scaled modes, which could be scaled in terms of unit modal mass and thus provide a complete modal model. This represents an advantage over the methods of Ref.[3], which give unscaled modes.

5. Conclusion

A method for updating a modal model on the basis of response measurements only has been extended from the case of a single impulsive excitation, to a general broadband excitation which does not have to be the only force as long as it is dominant. This dominant excitation does not have to be white, though its log spectrum should be reasonably smooth and flat so that its cepstrum is short. From limited data, it appears that the dominant force should be about four times larger than the next largest. The method can be used to update a modal model, which has for example been measured in the laboratory or perhaps estimated using a finite element model, to account for the expected minor changes caused by different operating conditions in service. Such updating would however be very valuable in cases where it is desired to use the modal information for the inverse problem of estimating excitation forces from measured responses, this being very sensitive to the accuracy of the modal data.

The method is based on principal components analysis by singular value decomposition, to separate out the principal response autospectra at each point corresponding to the dominant force, and on extracting the poles and zeros of each FRF from the cepstrum formed from the corresponding first principal autospectrum. Poles and zeros thus obtained are for the corresponding minimum phase FRF, but could be adjusted on the assumption that the poles will be minimum phase for a stable system, and that zeros will most likely have the same characteristics as the corresponding ones in the original measurements. For small changes it is assumed that a "scaling and equalisation function" can be carried over from the original measurements. This compensates for the effects on the shape of the FRFs of out-of-band modes which are not measured. The current results confirm that this is a reasonable assumption, as found in [2], provided that modal frequencies do not change by more than about 10%, and that mode shapes do not change greatly. The changes induced by the modifications to the frame reported here did not satisfy this latter requirement for higher modes than the first two, but the equalisation and scaling functions obtained from the updated FRFs could be used with the updated poles and zeros from the responses to regenerate reasonable scaled estimates of the measured FRFs over the full bandwidth. Damping values were not

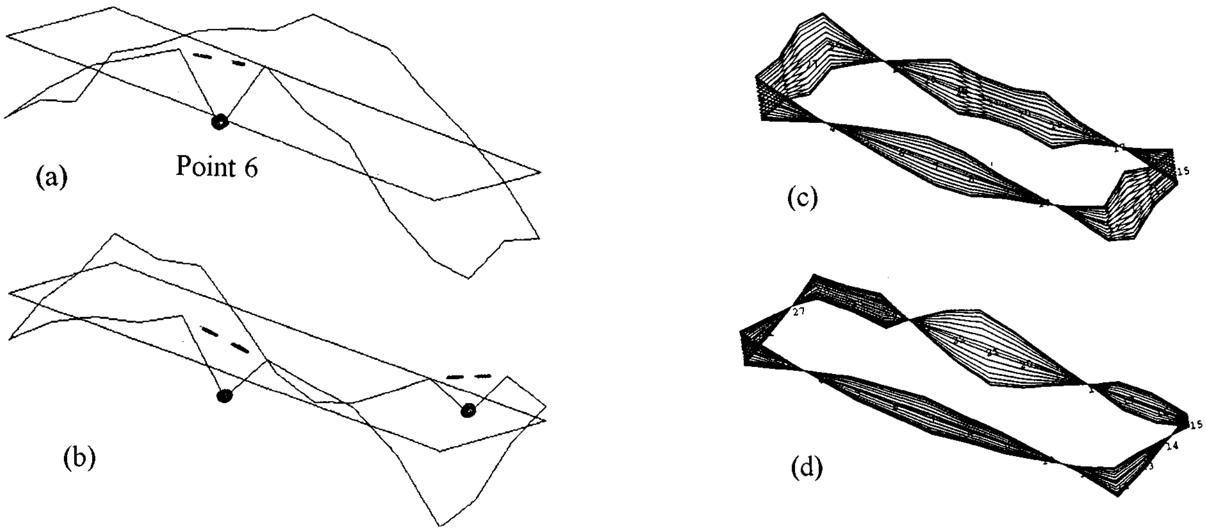

Figure 8 Comparison of mode shapes from regenerated FRFs (a, b) with those by direct measurement (c, d). (a) and (c) are for mode 2, (b) and (d) are for mode 10. Points with obvious faults are indicated.

estimated very accurately, but this is possibly due to the amount of noise in the extracted autospectra. It will be tested if this can be reduced by increasing the number of averages. It may be found preferable to generate all required cross spectra at the time of measurement rather than recording a smaller number of very long time records. For N measurement points it would be necessary to measure N2 cross spectra (or a little over a half this if symmetry is assumed) as against N time records, but on the other hand the latter might each have to contain several hundred times more data to allow sufficient noise reduction by averaging. Another possibility would be to smooth the cross spectra (from a limited number of averages) before singular value decomposition, although this may defeat the object by increasing apparent damping.

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