Output-only time-frequency-domain modal identification of time-varying structures using a recursive two-stage least square method

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Abstract
Real-time acquisition of modal parameters may contribute to the on-line structural dynamic research of time-varying structures, such as the health monitoring, the damage detection, the vibration control, etc. The recursive algorithms of modal parameter estimation supply the fundamental of acquiring modal parameters in real-time. This paper presents a recursive method of modal parameter estimation for time-varying structures, including the recursive estimation of time-dependent power spectra using the recursive least square filter, modal parameter estimation at each time instant using the least square complex frequency-domain method, the modal parameter validation using the recursive fuzzy clustering and the continuous-time estimation of modal parameters using a recursive least square approach. An experimental example validate the proposed method finally.

1 Introduction

In the real world, many engineering structures, such as traffic-excited bridges, launch vehicles with varying fuel mass, airplanes in flight with varying additional aerodynamic effects, deployable and flexible geometry-variable aerospace structures, rotating machinery and etc., show properties changing with time. In many real-life applications, the excitation on the time-varying structures is unknown and random so that operational or output-only methods are appropriate.

The currently existing methods of modal parameter estimation for time-varying structures can be classified into two categories: time-domain parametric methods and time-frequency-domain non-parametric methods.

In the past decades, many time-domain parametric approaches of dynamic identification for time-varying structures were presented. There are two categories of time-domain parametric identification approaches for time-varying structures: autoregressive moving average model based and state-space model based approaches. In the first category, Petsounis and Fassois [1, 2] presented the time-dependent autoregressive moving average (TARMA) representation for the modeling of non-stationary stochastic vibration. Poulimenos and Fassois [3] surveyed and compared several approaches of TARMA-based non-stationary random vibration modeling including unstructured parameter evolution, stochastic parameter evolution, and deterministic parameter evolution. Spiridonakos, Poulimenos, and Fassois [4, 5] estimated the modal parameters via the TARMA-based approaches and validated them with a laboratory experiment. In the second category, Liu [6, 7] proposed a state space based approach for linear time-varying systems via decomposing a series of Hankel matrices that are assembled by output response data or additional input data with singular value decomposition (SVD). Liu and Deng [8, 9] improved the state space based
approach for linear time-varying system through making it less sensitive to noise and validated the identification algorithm with an experiment of a moving cantilever beam.

On the other hand, several time-frequency analysis-based non-parametric identification approaches were developed in the past decade. Ghanem and Romeo [10] presented a wavelet-based identification approach, which transforms the classic governing equation of motion into a wavelet expanded form by projecting the physical responses to a series of wavelet coefficients, and identified the modal parameters by solving the expanded-form equation. Roshan-Ghias et al. [11] estimated modal parameters using smoothed pseudo Wigner-Ville distribution (SPWVD), which represents the analytical explicit responses onto WVD plots, and estimated the natural frequency and damping ratio of a SDOF system with tracking the ridge of these plots. Xu [12] presented an approach of modal parameter estimation through Gabor expansion of response signals. Meanwhile, some approaches using Hilbert transform (HT) or Hilbert-Huang transform (HHT) were proposed. Xu et al. [13] decomposed the responses into a series of single components with Gabor expansion and identified the modal parameters of these single-component signal with HT. Shi, Law and Xu [14] decomposed the responses with empirical mode decomposition (EMD).

In author’s previous work, the time-frequency-domain two-stage least square method of modal parameter estimation for time-varying structures is presented [15, 16]. This paper attempts to adapt the previous method to the recursive behavior for the potential applications of real-time estimation methods of modal parameters. The reminder of this paper is organized as follows. Section 2 introduces the basic idea of the recursive least square adaptive filter. Section 3 illustrates the fuzzy clustering and its recursive algorithm. Section 4 presents an experiment of a time-varying structure. Section 5 presents the recursive method of modal parameter estimation for time-varying structure which includes the recursive estimation of time-dependent power spectra using the recursive least square filter, modal parameter estimation at each time instant using the least square complex frequency-domain (LSCF) method, the modal validation using the recursive fuzzy clustering and the continuous-time estimation of modal parameters using a recursive least square approach. In Section 5, the proposed method is validated by the experiment of Section 4.

2 Recursive least square adaptive filter

In this paper, the recursive least square (RLS) adaptive filter is used to estimate time-dependent spectra of structural responses. This section introduces the derivations and the solution of the recursive least square adaptive filter.

A discrete-time non-stationary process \( x(n) \) can be modeled by a time-dependent autoregressive (TAR) based on the linear difference equation of order \( m \) [17] as,

\[
x(n) = -\sum_{k=1}^{m} a_{k}(n) x(n-k) + w(n)
\]

where \( w[n] \) is a stationary white noise process with zero mean and variance \( \sigma_w^2(n) \) and \( a_{k}(n) \) are the TAR parameters, which are found by the recursive least squares adaptive filter in this paper. When the TAR parameters are known, the prediction of the next sample of the process [18] is given by,

\[
\hat{x}(n) = -\sum_{k=1}^{m} a_{k}(n-1) x(n-k)
\]

In recursive implementations of the least squares, the cost function to be minimized is defined by,

\[
\ell(n) = \sum_{i=1}^{n} \beta(n,i) |e(i)|^2
\]

where the weighting factor \( \beta(n,i) \) is defined by the exponential forgetting factor as \( \lambda^{-i} \), in which \( \lambda \) is a positive constant close to, but less than, unity. The equation error is defined by,

\[
e(i) = x(i) - x(i) = x(i) - \sum_{k=1}^{m} a_{k}(n-1) x(i-k) = x(i) - a^\top(n)u(i)
\]
where \( a(n) = [a_1(n), a_2(n), \ldots, a_n(n)]^T \) and \( u(i) = [x(i), x(i-1), \ldots, x(i-m+1)]^T \).

To overcome the “ill-posed” nature of the recursive least squares, the regularization is necessary, so the regularized cost function is defined by,

\[
J(n) = \sum_{i=1}^{n} \lambda^{i-1} |e(i)|^2 + \delta \lambda^n |a(n)|^2
\]

(5)

where \( \delta \lambda^n |a(n)|^2 \) is a “rough” form of regularization and the normal equations can be written in matrix form as [19]

\[
\Phi(n)a(n) = z(n)
\]

(6)

with

\[
\Phi(n) = \sum_{i=1}^{n} \lambda^{i-n} u(n) u(i) + \delta \lambda^n I
\]

(7)

\( \Phi(n) \) and \( z(n) \) can be obtained by the recursive computations as

\[
\Phi(n) = \lambda \Phi(n-1) + \lambda^{n-n} u(n) u(i) \\
z(n) = \lambda z(n-1) + u(n) d(n)
\]

(8)

As a result, the TAR parameters \( a(n) \) can be updated by the recursive approach.

3 Fuzzy clustering and its recursive format

3.1 Fuzzy clustering

Fuzzy cluster analysis or fuzzy clustering partition a set of data into \( C \) clusters and each cluster is represented by its center, also called prototype. In other words, the aim of the fuzzy cluster analysis is to determine the prototypes and the memberships.

Let \( X = \{x_1, x_2, \ldots, x_N\} \subset \mathbb{R}^p \) be a data set with \( N \in \mathbb{N} \) data points. The memberships can be represented by a matrix \( U \in \mathbb{R}^{C \times N} \) with \( C \in \mathbb{N} \) clusters. An element in the membership matrix \( u_{ij} \), with \( i = 1, 2, \ldots, C \) and \( j = 1, 2, \ldots, N \) denotes the degree of the \( j \)-th data point belonging to the \( i \)-th cluster relative to other clusters. Because the membership matrix, \( U \), represents a probabilistic cluster partition, there are a few constraints on \( U \):

\[
\forall i \in \mathbb{N}^C_{(1,C)} \cap \mathbb{N}^N_{(1,N)}: \quad u_{ik} \in [0,1]
\]

(9)

\[
\forall j \in \mathbb{N}^N_{(1,N)}: \quad \sum_{i=1}^{C} u_{ij} = 1
\]

(10)

\[
\forall i \in \mathbb{N}^C_{(1,C)}: \quad 0 < \sum_{j=1}^{N} u_{ij} < N
\]

(11)

Eq.(10) and Eq. (11) imply that the total membership for the \( j \)-th data point to all \( C \) clusters is 1, and for each cluster, at least one data point belongs to that cluster with a nonzero membership, respectively.

Let \( V = \{v_1, v_2, \ldots, v_C\} \subset \mathbb{R}^p \) be a set of prototypes with \( C \) cluster prototypes. \( v_i \) is the prototype of the \( i \)-th cluster.

The fuzzy c-means (FCM) [20] algorithm is one of the objective-based fuzzy clustering algorithms. Given the data set \( X \), the membership matrix \( U \), the cluster prototypes \( V \) and the distance function \( d \), a scalar objective function is described by:

\[
J(X,U,V) = \sum_{i=1}^{C} \sum_{j=1}^{N} u_{ij}^m d^2(x_i, v_j)
\]

(12)
where $J \in \mathbb{R}^l$, the exponent $m \in \mathbb{R}_+$ is the fuzzy exponent and $d(\cdot, \cdot) \in \mathbb{R}^l$ denotes the distance between two vectors, which does not depend on the memberships. Commonly, $m = 2$ and $d(\cdot, \cdot)$ is the Euclidean distance function as:

$$d_{ij}^2 = d^2(x_j, v_i) = (x_j - v_i)^T (x_j - v_i)$$  \hspace{1cm} (13)

The fuzzy c-means algorithm (FCM) is one of the most popular algorithms of fuzzy cluster analysis. There are five phases in FCM:

Phase-I. Chosen the number of clusters $C$, the fuzzy exponent $m$, the convergence criterion $\zeta$ and the maximum number of iterations $M$, the initial membership matrix $U^{(0)}$ is defined randomly, which satisfies the constraints as Eq. (9), (10) and (11).

Phase-II. Update the cluster prototypes, $V^{(k)}$, at the $k$th iteration by

$$v^{(k)}_i = \frac{\sum_{j=1}^{C} u_{ij}^{(k-1)} x_j}{\sum_{j=1}^{C} u_{ij}^{(k-1)}}$$  \hspace{1cm} (14)

Phase-III. Calculate the distances $d_{ij}^{(k)}$ for all $i = 1, 2, ..., C$ and $j = 1, 2, ..., N$ using the updated cluster prototypes, $V^{(k)}$, based on the distance function as defined in Eq. (13) or other distance functions.

Phase-IV. Update the membership matrix, $U^{(k)}$, at the $k$th iteration by

$$u_{ij}^{(k)} = \begin{cases} \frac{1}{\sum_{r \in \Gamma_j} \left( d_{ij}^{(k)}/d_{ij}^{(k)} \right)^{2/(m-1)}} \text{, for } \Gamma_j = \varnothing \\ \sum_{r \in \Gamma_j} u_{ij}^{(k)} \text{, for } \Gamma_j \neq \varnothing, j \in \Gamma_j \\ 0 \text{, for } \Gamma_j \neq \varnothing \text{, } j \notin \Gamma_j \end{cases}$$  \hspace{1cm} (15)

where $\varnothing$ is the null set and $\Gamma_j = \{ i \mid d_{ij}^{(k)} = 0 \}$. If $\Gamma_j$ contains more than one element, $u_{ij}^{(k)}$ for $r \in \Gamma_j$ is not uniquely determined and more operations are needed [20].

Phase-V. Check the termination criteria including the convergence criterion $\zeta$ and the maximum number of iterations $M$. If $\|U^{(k)} - U^{(k-1)}\| < \zeta$ or $k > M$, the iteration terminates; otherwise, repeat phase-II, III and IV.

A data point coinciding with a prototype causes a singularity in the FCM algorithm. In the beginning of phase IV of the FCM algorithm, singularity is checked and a set of the indices of the singular data points, $\Gamma_j$, as shown in Eq. (15), is formed.

### 3.2 Recursive fuzzy clustering

When the observed data changes with the time, the recursive clustering is necessary to capture the current features of the new-coming data. A approach of recursive fuzzy clustering is introduced as follows [21].

The $i$th cluster prototype at the time instant $n$ is defined by $v_i(n)$. The relation between the old cluster prototype and a new one can be expressed by,

$$v_i(n+1) = v_i(n) + \Delta v_i(n+1)$$  \hspace{1cm} (16)

with the increment,
As shown in Eq. (17), the calculation of the membership at the \((n+1)\)th time instant requires the past \(n\) memberships, which is against the recursive idea. Therefore, an approximate calculation of the denominator of Eq. (17) is achieved by adding the forgetting factors for the past memberships as defined as follows,

\[
s_i(n+1) = \gamma s_i(n) + u^m_{i(n+1)}
\]

where \(\gamma\) is the forgetting factor and \(0 < \gamma \leq 1\). As shown in Eq. (18), the denominator at the time instant \(n\), \(s_i(n+1)\), can be calculated by a recursive approach. Furthermore, the current membership is defined by,

\[
u^m_{i(n+1)} = \left( d^2 \sum_{j=1}^{C} \left( 1/d^2_{ij(n+1)} \right)^{\gamma/j} \right)^{-1}
\]

with the distance function defined as Eq. (13).

\section{Experiment}

This section presents an experiment of a beam-like time-varying structure as well as its corresponding test and modal parameters of the baseline models.

\subsection{Setups}

Fig. 7(a) shows the schematic diagram of the experimental structure and its laboratory setup. The structure consists of a steel beam and a steel mass, of which the property parameters are shown in Table 1. The structure is vertically suspended by flexible rubber ropes and tied up by nylon cords in the horizontal plane. The nylon cords only constrain rigid motion of the beam in the horizontal plane and not its vibration in vertical direction. The mass can slide along the axial direction of the beam driven by hand from the center to 500mm away. Eleven piezoelectric accelerometers measure the acceleration responses of the beam at eleven uniformly distributed positions along the axial direction of the beam. A laser distance meter (measuring distance: 100mm-600mm) measures the instantaneous position of the mass. A LMS\textsuperscript{TM} SCADAS III system acquires the acceleration response signals conditioned by a conditioner, the position signals of the mass and the force signal of the impact hammer. The PC records the input signals via the SCADAS system and LMS Test.Lab software. Fig. 7(b) shows a photo of the structure and the laboratory setup for hammer impact testing.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{image}
\caption{Schematic diagram of the experimental structure and its laboratory set-up}
\end{figure}
### 4.2 Baseline results

The time-varying structure in this paper is also considered as a linear parameter-varying (LPV) structure. The structural dynamic characteristics are functions of the position of the mass; that is, while other parameters are fixed; the position of the mass could be continuous-time-varying. In other words, when the mass stays at a fixed position, the structural dynamic characteristics, such as modal parameters, can be obtained through the available time-invariant system identification approaches. Therefore, when the mass is fixed, the structure is in a fixed configuration[4]. The fixed-configuration structure is the baseline model for the time-varying structure. Moving the mass from the center of the beam to 500mm away with a spacing of 5mm, 101 baseline models can be obtained. The acceleration signals and the force signal of the hammer are measured with a sampling frequency of 512Hz and sampling time of 2s. In order to estimate the modal parameters of the baseline models, the least squares complex exponential method (LSCE) in LMS Test.Lab is used. Figure 2(a) and (b) show the resonance frequency and damping ratio (“○”: mode 1; “+”: mode 2; “×”: mode 3; “*”: mode 4), respectively.

![Figure 2: Resonance frequency and damping ratio of the baseline models](image)

#### 4.3 Measurements of the time-varying structure

In this section, the structure is time-varying, because the mass slides continuously. The setup is the same as that for the fixed-configuration structure. The sampling frequency is 512Hz and the record length is 8s. Figure 3 shows the position of the mass at each time instant. Before 1.044s the mass did not begin to move and after 7.626s the distance between the mass and laser meter was over the range of the laser meter.

![Figure 3: The motion of the mass](image)
The hammer impacts the time-varying structure intermittently during the measurement and the impacts begin before recording the data. The 11 output responses are measured by the accelerometers and four of them, Output 1, 4, 7 and 11, are shown in Figure 4. The time signals of the 11 output responses are the basic data for the following modal parameter estimation.

![Figure 4: Responses for Output 1, 4, 7 and 11](image)

5 **Recursive two-stage least square estimator for time-varying structures**

5.1 **Procedure of the estimation**

The procedure of the recursive two-stage least square estimator is shown in Figure 5. The first phase recursively estimates the time-dependent power spectra (PSD) of the current time instant using the recursive least square (RLS) filter. Based on the time-dependent power spectra, the second phase estimates the modal parameters of the current time instant using the LSCF estimator [22]. The third phase validates the estimated modal parameters based on the recursive fuzzy clustering introduced in Section 3. Finally, the continuous-time-varying modal parameters are estimated by a recursive least square approach recursively.

![Figure 5: Procedure of the estimation](image)

The estimator is named recursive two-stage least square estimator, because it includes two least-square stages and each phase involves the structural responses before the current time instant and has the recursive behaviour.
5.2 Estimation of PSDs using the recursive least square filter

The recursive least square filter has been introduced in Section 2. After the RLS filter estimates the TAR parameters, the PSDs can be calculated as,

\[ P(n, f) = \frac{\sigma_n^2(n)}{1 + \sum_{i=1}^{N_n} a_i(n) e^{-j2\pi n T_s}} \]

(20)

Assume that the variance \( \sigma_n^2(n) \) is unity, the PSDs for the Output 1, 4, 7 and 11 are shown in Figure 6.

![Figure 6: PSDs for Output 1, 4, 7 and 11](image)

5.3 Modal parameter estimation using the LSCF

At each time instant, the PSD are parametrically defined by a common-denominator model as

\[ G(\omega) = \frac{B(\omega)}{A(\omega)} \]

(21)

where \( B(\omega) \) is the numerator polynomial and \( A(\omega) \) is the common-denominator polynomial as follows

\[ B_i(\omega) = \sum_{l=0}^{N_i} \Omega_l(\omega) B_{ij} \]
\[ A(\omega) = \sum_{l=0}^{N_N} \Omega_l(\omega) A_i \]

(22)

where \( B_i(\omega) \) is the numerator polynomial of \( i \)-th reference point and \( j \)-th responses point ; \( \Omega_l(\omega) \) are the \( l \)-th order polynomial basis functions and \( N_i \) is the order of the polynomials. In the least square complex frequency-domain (LSCF) approach, \( \Omega_l(\omega) \) is commonly defined as a z-domain variable with sample period \( T_s \):

\[ \Omega_l(\omega) = e^{-i\omega l T_s} \]

(23)

The polynomial leading coefficients, \( B_{ij} \) and \( A_i \), assemble the unknown parameters, \( \theta \), in a least square cost function as follows:
where the equation errors are represented as

$$
\varepsilon^L_S = W_0(\omega_j)(B_0(\omega_j, \mathbf{\beta}_j) - A(\omega_j, \mathbf{\alpha})G_0(\omega_j))
$$

(25)

where \( \mathbf{\beta}_j \) and \( \mathbf{\alpha} \) are coefficient vectors containing \( B_{ijkl} \) and \( A_l \) for numerators and common-denominator respectively. Reference [23] described the details how to minimize the cost function defined in Eq. (24). Furthermore, the modal frequency and damping ratio can be calculated by the coefficient vector \( \mathbf{\alpha} \). The estimated modal frequency and damping ratio are shown in Figure 7.

5.4 Modal parameter validation using the recursive fuzzy clustering

This phase validates the estimated modal frequency as shown in Figure 7 using the recursive fuzzy clustering introduced in Section 3. The clustering results are shown in Figure 8, in which the letters “A”, “B”, “C” and “D” denote the four clusters of the modal frequency, the color of these letters indicates the memberships of the modal frequency and the black dots show the prototypes of these clusters. The prototypes of the clusters move from left to right due to recursive behavior of the recursive fuzzy clustering algorithm.
Furthermore, the memberships will be considered as the weights in the next continuous-time estimation of modal parameters using a recursive least approach. The validated modal parameters are expressed into the four sub-partitions with the corresponding memberships by

\[ \forall i \in \mathbb{N}_{1:C}, \forall j \in \mathbb{N}_{1:N_i}, N_i = |R_i|; \]
\[ R_i = \{x_{i,1}, x_{i,2}, \ldots, x_{i,L}, \ldots, x_{i,N_i}\} \subset \mathbb{R}^r, \]
\[ u_i = \{u_{i,1}, u_{i,2}, \ldots, u_{i,j}, \ldots, u_{i,N_i}\} \subset \mathbb{R}^l \]

where \( x_{i,j} \) is general data point, which can be the modal frequency or the damping ratio, \( u_{i,j} \) is the corresponding membership and \( N_i \) is the length of the \( i \)th sub-partition.

### 5.5 Continuous-time estimation of modal parameters using a recursive least approach

In this phase, the continuous-time-represented modal parameters are estimated by a recursive least square approach based on the validated modal parameters at all discrete time instants. The cost function of this recursive least squares is defined by

\[ \epsilon_i^{WLS}(\theta) = \sum_{j=1}^{N} \epsilon_i^{WLS}(t_j, \theta)^2 \]

where \( i, j \) and \( N \) are defined in Eq. (26) and with the equation error

\[ \epsilon_i^{LS}(t_j, \theta) = w_{i,j} (x_i(t_j) - x_{i,j}) \]

where \( w_{i,j} \) are the weights, \( x_{i,j} \) is an arbitrary component in \( x_{i,j} \) and the predict function \( x_i(t_j) \) is expressed by a polynomial as

\[ x_i(t) = \sum_{k=1}^{K} \theta_{i,k} p_k(t) \]

where \( \theta_i = \{\theta_{i,k} | k \in \mathbb{N}_{1:L}\} \) and the basis function \( p_k(t) \) are polynomials.

The estimation of the projection coefficients \( \theta \) can be obtained by minimizing the cost function, Eq. (27), as

\[ \hat{\theta} = \arg \min \{\epsilon_i^{WLS}(\theta)\} \]

where “arg min” means the “argument minimization”.

According to Eq. (28) and Eq. (29), the equation errors can be rewritten by

\[ \tilde{e}_i^{WLS} = \begin{bmatrix} w_{i,1} \left[ p_{i,1}(t_1) \right] & \left[ p_{i,2}(t_1) \right] & \ldots & \left[ p_{i,L}(t_1) \right] \\ w_{i,2} \left[ p_{i,1}(t_2) \right] & \left[ p_{i,2}(t_2) \right] & \ldots & \left[ p_{i,L}(t_2) \right] \\ \vdots & \vdots & \ddots & \vdots \\ w_{i,N_i} \left[ p_{i,1}(t_{N_i}) \right] & \left[ p_{i,2}(t_{N_i}) \right] & \ldots & \left[ p_{i,L}(t_{N_i}) \right] \end{bmatrix} \begin{bmatrix} \theta_{i,1} \\ \theta_{i,2} \\ \vdots \\ \theta_{i,L} \end{bmatrix} = \tilde{P} \theta - \tilde{R} \]

As defined in Eq. (31), the least square problem is linear. Hence, the projection coefficients, \( \theta_i \), can be estimated [24] as follows:

\[ \hat{\theta} = (\tilde{P}^T \tilde{P})^{-1} \tilde{P}^T \tilde{R} \]
The \( \hat{\Theta} \) can be estimated as a recursive behavior. Define the the projection coefficients of the \( n \)th time instant \( \hat{\Theta}(n) \), the \( \hat{\Theta}(n+1) \) can be calculated by

\[
\hat{\Theta}_{n+1} = \left( \hat{P}_n^T \hat{P}_n \right)^{-1} \hat{P}_n^T \hat{R}_n = \left( \hat{P}_n^T \hat{P}_n + \hat{a}_{n+1}^T \hat{a}_{n+1} \right)^{-1} \left( \hat{P}_n^T \hat{R}_n + \hat{a}_{n+1}^T \hat{b}_{n+1} \right)
\]  

(33)

with

\[
\hat{P}_{n+1} = \begin{bmatrix} \hat{p}_n \end{bmatrix}, \quad \hat{R}_{n+1} = \begin{bmatrix} \hat{R}_n \end{bmatrix}, \quad \hat{a}_{n+1} = w_{n+1} \begin{bmatrix} p_1(t_{n+1}) & p_2(t_{n+1}) & \cdots & p_L(t_{n+1}) \end{bmatrix}, \quad \hat{b}_{n+1} = w_{n+1} x_{n+1}
\]  

(34)

In this recursive least squares, the highest order polynomial \( L \) is 7, and the weights, \( w_i = \left\{ w_{i,j} \mid j \in \mathbb{N}^{[1,\lambda_i]} \right\} \), are defined by \( w_{i,j} = u_{i,j}^{\kappa} \) with \( \kappa = 5 \). The continuous-time estimated modal frequency and damping ratio are shown in Figure 9.

![Figure 9: Estimated modal frequency and damping ratio](image)

(a) Resonance frequency   
(b) Damping ratio

As shown in Figure 9, the continuous-time estimation of the modal frequency coincides with the results of the baseline models very well and that of the damping ratio is at the same level as the results of the baseline models. Because the proposed continuous-time estimation is recursive, the computational cost of this phase is very low.

### 6 Conclusion

Based on the knowledge of the recursive least square filter, the least square frequency-domain method of modal parameter estimation and the recursive fuzzy clustering, this paper presents an output-only time-frequency-domain two-stage least square estimator of modal parameters for time-varying structures. In addition, an experimental example has validated this estimator. Because of the recursive ability of each phase of the estimator, the proposed estimator has the recursive behavior and has the low computational cost, which indicates that the proposed estimator is potential to be applied in the real-time estimation of modal parameters for time-varying structures.

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