Inputshaping: a linear programming approach

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Abstract

Input shaping is an established technique to generate prefilters that move flexible mechanical systems with little or no residual vibration. Traditional input shaping design strategies are often analytical; however, the present paper introduces a design method based on numerical optimization. The paper shows that, through a careful selection of the optimization variables, objective function and constraints, it is possible to obtain a linear optimization problem. As a result, it is guaranteed that the globally optimal input shaper be found in a few seconds of computational time. The presented optimization framework is able to handle higher-order, linear time-invariant dynamic systems, as opposed to traditional input shapers, which are mainly based on second-order systems. Moreover, constraints on input, output and state variables are easily accounted for, as well as robustness against parametric uncertainty. Numerical results illustrate the capability of the proposed design approach to reproduce existing input shaping design approaches, while experimental results illustrate its potential for higher-order systems.

1 Introduction

Input shaping is an established technique to generate finite impulse response (FIR) prefilters for flexible mechanical systems. These FIR-filters convert the reference point-to-point motion commands such that very little or no residual vibration occurs upon arrival at the endpoint. That is, the system stays fixed at the end position. The residual vibration is characterized by the exponential envelop of the vibration. The price to be paid for the residual vibration suppression is a short delay, known as move-time penalty, equal to the duration of the prefilter's impulse response.

Hence, the basic idea of input shaping is that the input shaping FIR-filter F converts the desired motion r to an input u. This input drives the system P, of which the output is denoted by y. The objective of the input shaping design is to determine the number K, the time locations t_k and the amplitudes f_k of the impulses of F, such that the output y does not exhibit residual vibration.

Singer and Seering [1] developed the first input shaping design approach. This approach is based on analytic expressions for the response of a continuous-time second-order system to the following sequence of $K+1$ impulses at time locations t_k:

$$f(t) = \sum_{k=0}^{K} f_k \delta(t - t_k). \tag{1}$$

The amplitude of the resulting vibration at t_K is given by

$$A_{res} = \sqrt{\left(\sum_{k=0}^{K} B_k \cos\phi_k\right)^2 + \left(\sum_{k=0}^{K} B_k \sin\phi_k\right)^2}, \tag{2}$$

where

$$\phi_k = \omega_n \sqrt{(1-\zeta^2)}t_k \tag{3}$$

$$B_k = \frac{f_k \omega_n}{\sqrt{(1-\zeta^2)}} e^{-\zeta\omega_n(t_K - t_k)}. \tag{4}$$

These expressions only depend on the undamped natural eigenfrequency ω_n and the damping ratio ζ of the system. From (2) and (3)–(4), it follows that zero vibration at time t_K, i.e., $A_{res} = 0$, is obtained provided that:

$$\sum_{k=0}^{K} f_k \mathrm{e}^{-\zeta\omega_n(t_K - t_k)} \sin(t_k \omega_n \sqrt{1-\zeta^2}) = 0 \tag{5}$$

$$\sum_{k=0}^{K} f_k \mathrm{e}^{-\zeta\omega_n(t_K - t_k)} \cos(t_k \omega_n \sqrt{1-\zeta^2}) = 0. \tag{6}$$

The minimum number of positive impulses, required to solve (5)–(6), is two: one at $t_0 = 0$ and one at $t_1 = T_0/2$, where T_0 denotes the period of the damped natural eigenfrequency. Multiple solutions for these equations exist as this problem is periodic. However, the ideal solution is determined based on the introduced move time penalty. The later the last impulse, the bigger the move time penalty and hence the bigger the introduced delay for the system. Furthermore, this basic prefilter design is very sensitive to ω_n and ζ: small errors on these parameters can yield large residual vibrations and hence an ineffective prefilter.

Robust design approaches have been developed to cope with this problem. A first robust approach is the *zero vibration derivative* (ZVD) approach [1] which results in first-order sensitivity robustness. This robustness is obtained by imposing, on top of (5)-(6), that the derivatives of these equations with respect to ω_n and ζ be zero. An other approach is the *extra insensitive* (EI) approach [2]. This approach does not require zero vibration in the nominal system. Alternatively, it is imposed that two systems that are slightly different from the nominal system, be without vibrations. A disadvantage of these and other [3] robust design approaches is that the additional robustness comes at the cost of extra move-time penalty.

The above mentioned input shaping techniques are used in industry for applications involving cranes [4], positioning of cartesian machines [5], mine-detecting robots [6], ... There are also numerous extensions on the basic technique. These are e.g. extensions for multi-input systems [7], inclusion of a priori information of the reference trajectory r to make the input shaper more efficient [8], ...

The classical techniques rely on nonlinear analytical expressions, such as (5)–(6), for the residual vibration of second-order systems. The basic equations are therefore solved analytically. Extensions to higher-order systems are possible [9], but result in suboptimal (i.e., with long move-times) prefilters and/or complex expressions that are tedious to derive. Also, several extensions are possible, but these lead often to nonlinear problems or approximate solutions as e.g. [10] for the inclusion of negative impulses and [11] for the inclusion of input constraints.

Therefore, this paper introduces a design method based on numerical optimization. It is shown that by using a proper reformulation of the input shaping problem it is possible to obtain a linear optimization problem (or linear program LP). This framework is applicable to any single-input single-output linear time invariant system of arbitrary order, and system uncertainty as well as constraints on system input and output can be accounted for. As a result, it is guaranteed that the globally optimal input shaper be found in a few seconds computational time. This approach overcomes some of the main difficulties that would occur if the input shaper design were formulated as a nonlinear optimization problem. In the latter case, the optimization algorithm is likely to get stuck in a local optimum, yielding suboptimal results, while it is impossible to guarantee that the global optimum has been found.

The paper is organized as follows. Section 2 describes the developed linear programming framework to design optimal input shaping filters and relates this approach to classical analytical methods. Section 3

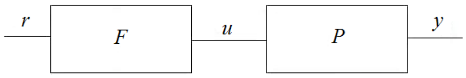

Figure 1: Overview of the considered systems. The prefilter F converts a reference r to the input u which leads to output y of system P.

validates the developed framework numerically, by reproducing the results as presented in [1] and [2]. Section 4 discusses the experimental validation of the developed framework on a 2 degree-of-freedom (DOF) mass-spring-damper system, and illustrates the importance of a robust design.

In this paper we assume that the system is unit scaled, i.e., the maximum input and output are scaled such that this maximum value is one.

2 Linear Programming Framework

An input shaping FIR prefilter F transforms a desired point-to-point motion r into an input u of the system P. When applied, this input causes an output y of the system, which ideally has no or little residual vibrations. (see fig. 1)

Designing a FIR filter leads to determining the number K, the moments t_k and the amplitudes f_k of (1). In the basic input shaping methods, this is accomplished by solving (5)-(6) simultaneously.

One approach to solve this problem could be to fix the number of impulses K, and solve for the t_k and f_k of the FIR-filter. However, this leads even for the basic input shaping problem to a nonlinear optimization problem which yields suboptimal filter design due to local minima. The nonlinearity arises of the combination of f_k and t_k in the vibration behavior, see (5)–(6).

The nonlinear dependency of (5)–(6) form the design parameters is avoided by fixing the moments t_k of the inputs of the prefilter. This approach is followed in this paper. The moments t_k of the impulses of the filter are fixed, and the amplitudes and total number of pulses are determined by mean of numerical optimization. This approach yields that the constraints for zero residual vibration become linear in the remaining design variables f_k, which allows to recast the optimization problem as an LP. This LP can be solved very efficiently with a guarantee of optimality. In order to achieve this, the considered objective and constraint functions considered must be linear in f_k. Subsection 2.1 discusses this LP formulation of the input shaping problem in detail. By providing a dense grid of possible FIR filter impulse moments t_k, this approach will not impose any practical performance limitations on the resulting filter. A moment t_k is called active if the corresponding f_k value is different from zero. The aim of the LP is to find a filter with as few active moments as possible that satisfies the constraints and optimizes the objective function. Another important feature of the presented framework is its time optimality, that is, the move time of the resulting input shaping filters, which is determined by the duration of this FIR filter, is minimal. Subsection 2.2 discusses how time optimality is achieved. Subsection 2.3 discusses extensions of the basic design framework to account for input and output constraints, and system uncertainty.

2.1 Rationale of the developed method

The linear programming framework is based on generalizing conditions (5)–(6), which are valid for second-order systems only. This generalization is based on the following expression of the output $y(t)$ of an LTI system described by its impulse response $p(t)$:

$$y(t) = r(t) \otimes f(t) \otimes p(t), \tag{7}$$

where $\otimes$ denotes the convolution operator. $f(t)$ and $r(t)$ denote the impulse response of the input shaping prefilter F and the reference trajectory respectively. Hence, if $r(t)$ is constant for $t \geq t_v$, then the output $y(t)$ of the system is constant for $t \geq t_K + t_v$ if:

$$g(t) = f(t) \otimes p(t) = 0 \text{ for } t \geq t_K \tag{8}$$

where $g(t)$ is the impulse response of the combination of prefilter and system. This condition for zero residual vibration is applicable to any LTI system of arbitrary order provided that its impulse response is available. Hence, (8) generalizes (5)–(6) and is the basic condition that needs to be fulfilled by filter F.

By considering a discrete-time description of the system at a selected sampling period T_s and by providing an equidistant grid of possible prefilter impulse moments at multiples of the sampling period T_s, the formulation of the input shaping prefilter design as a numerical optimization can be simplified considerably. Here, a discrete state space model is preferred as this allows to impose easily optional constraints on the state variables:

$$x_{k+1} = Ax_k + Bu_k \tag{9}$$
$$y_k = Cx_k + Du_k. \tag{10}$$

The discrete-time formulation of the FIR input shaping prefilter (1) with $t_k = kT_s$ equals:

$$f(k) = \sum_{i=0}^{K} f_k q^{-i} \tag{11}$$

with q^{-1} the backward-shift operator.

Based on (9)–(10) and (11), it follows that the zero initial state impulse response g_k of P and F together can be written as:

$$x_{k+1} = Ax_k + Bf_k, \tag{12}$$
$$g_k = Cx_k + Df_k, \tag{13}$$

which is the response of system P excited by the impulse response of the prefilter F. This formulation of (12)–(13) for $k = 0, 1, ...K$ can be reformulated as a homogeneous set of linear equations:

$$\mathbf{E}\tilde{\mathbf{x}} = \mathbf{0}, \tag{14}$$

where $\tilde{\mathbf{x}} \in \mathbb{R}^{(n+2)(K+1)}$ is a vector containing $K + 1$ filter amplitudes $f_k = f(t_k)$, $K + 1$ total impulse response amplitudes $g_k = g(t_k)$ and the state vectors $x_{k+1} = x(t_{k+1})$ for $k = 0, \ldots, K$, with n the order of the system. The matrix $\mathbf{E} \in \mathbb{R}^{(n+1)(M+1)\times(n+2)(K+1)}$ represents the system dynamics. Matrix $\mathbf{E}$ is a sparse matrix with a block matrix structure.

An alternative and more condensed formulation of the zero initial state response g_k is obtained from (14) by eliminating all state variables, yielding:

$$\mathbf{Hf} = \mathbf{g}, \tag{15}$$

with $\mathbf{H} \in \mathbb{R}^{(M+1)\times(K+1)}$ a lower triangular Toeplitz matrix containing the dynamics of the system. $\mathbf{f}$ and $\mathbf{g} \in \mathbb{R}^{K+1}$ contain the elements of the discrete-time impulse response of the prefilter and system-prefilter combination, respectively. This more condensed formulation (15) yields a reduced number of equality constraints in the LP formulation of the input shaper design at the cost of a worse conditioned problem.

This representation allows to deduce easily the required number of constraints on g_k to ensure zero vibration. Based on (15) and the theorem of Cayley-Hamilton [12], it is straigthforward to prove that the required number of constraints on g_k is n and hence $M = K + n$. That is, if $g_k = 0$, for $k = K + 1 \ldots K + n$, it is also so for all bigger k. With regard of expression (8), that means that $t_K = KT_s$. This is consistent with the fact that the basic input shaping system imposes two requirements for a second order system [1]. This set of constraints is obviously also valid if the sparse reformulation (14) is used.

Based on another derivation, it can also be proven that the last non-zero impulse of the prefilter, is at the latest at the moment that the total impulse response reaches its vibrationless mode. It is, if $g_k \equiv 0,$ for $k = K+1\ldots\infty$, then is $f_k \equiv 0,$ for $k = K+1\ldots\infty$ This extra condition has logically to be fulfilled, as the considered system is causal.

Because the condensed problem formulation (15) is often much worse conditioned than the state based problem formulation (14), the latter problem formulation (14) is used for the remainder of the paper.

The above discussion results in the basic optimization problem for input shaping:

$$\min_{f_k, g_k, x_k} \quad h(f_k) \tag{16a}$$

$$\text{subject to} \quad \mathbf{E}\tilde{\mathbf{x}} = \mathbf{0} \tag{16b}$$

$$g_k = 0 \text{ for } k = K+1\ldots K+n \tag{16c}$$

$$f_k = 0 \text{ for } k = K+1\ldots K+n \tag{16d}$$

$$\sum_{k=0}^{K} f_k = 1. \tag{16e}$$

Constraint (16e) is introduced without loss of generality to avoid the trivial solution $f_k \equiv 0$. However, in order to attain the same displacement as prescribed by the reference motion $r(t)$, this value should be chosen equal to one. The function $h(f_k)$ denotes the objective function. If h is linear in f_k, (16a)–(16e) constitutes an LP, since (16b)–(16e) are linear in the design parameters.

As sparse solutions are often preferred, that is, solutions in which many f_k are zero, the objective function $h(f_k)$ can be accommodated to promote sparsity. The one-norm is well-known [13] to have such an effect, and can be formulated such that it is linear in the optimization variables:

$$h(f_k) = \sum_{k=0}^{K} |f_k|. \tag{17}$$

If no $h(f_k)$ is specified, standard LP solvers consider (16a)–(16e) as a *feasibility problem*. That is, the solver assumes $h(f_k) = 0$ and just tries to find a solution f_k that complies with constraints (16b)–(16e). By solving a sequence of such feasibility problems, it is possible to minimize K, as explained in the next section.

2.2 Time optimality

In order to minimize the move time penalty introduced by the input shaper, t_K needs to be minimized. Given that $t_K = KT_s$, it follows that K should be minimized. Hence, the purpose is to find the lowest K, denoted as K^*, such that the corresponding filter still complies with (16b)–(16e).

K^* is found by solving, for various values of K, the LP (16a)–(16e) as a feasibility problem, i.e., with a goal function $h(f_k) = 0$. If the LP solver finds a solution f_k for the selected value of K, $K^* \leq K$. If the LP solver is not able to find a solution f_k for the selected value of K, i.e., if the problem is infeasible, it is obvious that $K^* \geq K$.

In order to minimize the required number of trials for K, a bisection algorithm is used. The bisection starts with a lower bound K_l (e.g. $K_l = 0$) and an upper bound K_u on K^* and proceeds as follows [13]:

repeat 1. $K = (K_u + K_l)/2$

2. solve the LP (16a)–(16c) with $h(f_k) = 0$

3. **if** the LP is feasible: $K_u := K$; **else** $K_l := K$

until $K_u - K_l \leq \epsilon = 0.5$

Exactly $\ulcorner \log_2((K_u - K_l)/\epsilon) \urcorner$ steps are required before the algorithm terminates. As only integer values are possible for the length of the filter, ϵ is chosen equal to 0.5.

The numerical results of Sec. 3 illustrate the effectiveness of this approach by showing that the framework is able to reproduce known analytical results.

2.3 Extensions

Input saturation can be avoided by imposing that the prefilter contains only positive impulses [1]. This constraint is a sufficient condition to avoid saturation if the reference motion is a unit step, but fairly conservative for smooth motion trajectories and then yields unnecessarily long move times. A more appropriate approach, yielding in general shorter move times, is to allow positive and negative filter impulses and imposing, similar to [10] and [11] the following constraint:

$$- |U| \leq \sum_{k=0}^{i} f_k \leq |U| \quad i = 0, \ldots, \tag{18}$$

where U is the maximum allowed input u. This requirement is sufficient for most point-to-point motions [10] and also necessary if this motion is a unit step. It is linear in the optimization variables.

Constraints on the output of the states can analogously be introduced. The overshoot of the system can e.g. be limited by imposing a constraint on the impulse response of the total system:

$$\sum_{k=0}^{i} g_k \leq 1 + \delta/100 \quad i = 0, \ldots, \tag{19}$$

where δ is the allowed overshoot. This constraint is necessary and sufficient if the reference input is a unit step, but only sufficient and hence conservative for other dwell-rise-dwell inputs.

The framework can also be extended to account for uncertainty of the system dynamics provided that a set of possible models Θ is available. This robustness is introduced by considering a finite representative set of system models θ_v from Θ in the prefilter optimization instead of only a nominal model. In addition, the zero residual vibration constraint 16c is relaxed to a finite bound γ on the vibration:

$$1 - \gamma \leq \sum_{k=0}^{i} g_k(\theta_v) \leq 1 + \gamma; \quad i = K + 1 \ldots M + 1; \quad \forall \theta_v \in \Theta. \tag{20}$$

As γ diminishes to zero, the zero vibration condition is approximated, but this results in longer filters.

Note that the introduced extra constraints are usually conservative. If a priori information about the reference trajectory is available, this can be introduced in these extra constraints to make them less conservative and hence the resulting prefilter shorter. This should be applied with care though, to avoid unwanted overshoot, vibrations, ... if the reference trajectory would be harder than expected.

If more extra constraints are added to the basic problem, there is often some extra freedom for choosing the ideal filter. Due to the discrete framework, there are multiple 'ideal length' filters, as the real 'ideal length' filter is only approximated from above. This allows/requires an extra goal function. To evoke sparsity, a goal function as (17) is imposed in combination with the extra constraints.

3 Numerical validation

This section validates the developed input shaping framework by comparing this framework with the input shaping approaches presented in [1] and [2]. The considered test case is a continuous-time second-order system:

$$P(s) = \frac{\omega_n^2}{s^2 + 2 \cdot \omega_n \cdot \zeta \cdot s + \omega_n^2}, \tag{21}$$

with $\omega_n = 2\pi$ rad/s and $\zeta = 0.5$. According to [1], the optimal prefilter, assuming positive impulses only, consists of two impulses ($K = 1$):

$$t_0 = 0 \quad f_0 = \frac{1}{1+R} = 0.86, \tag{22}$$

and

$$t_1 = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}} = 0.5773 \quad f_1 = \frac{R}{1+R} = 0.14, \tag{23}$$

where

$$R = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}}. \tag{24}$$

To reproduce these results with the developed input shaping framework, the system dynamics are first discretized for a certain sampling period T_s using the ‘zero-order-hold’ (ZOH) equivalent. The maximum number of prefilter impulses is set equal to $K + 1 = 2000$.The developed framework reproduces the prefilter obtained with the method of Singer and Seering [1], that is equations (22)–(24), exactly if t_1 (23) is an integer multiple of T_s, which is the case for $T_s = 0.00115s$. Fig. 2 shows the results for $T_s = 0.001s$ which is not an integer multiple of t_1. Three impulses instead of two are obtained. One at $t_0 = 0s$, with an amplitude exactly equal to f_0 (22), and two impulses at discrete time locations just before and after t_1. The sum of the amplitudes of these two impulses equals exactly f_1 (23).

This linear program is modeled in `matlab` and solved using `Mosek` [14]. The total CPU time required to calculate the prefilter is $18s$ on a Mobile Pentium 2GHz processor with 1GB RAM.

Next, the basic input shaping design is robustified for uncertainty on the undamped natural eigenfrequency ω_n. Numerical simulation shows that the developed framework can imitate the robust ZVD filter [1] and EI filter [2] designs. The ZVD filter design uses a local sensitivity approach to include robustness, i.e. robustness is obtained by setting the derivative of the amplitude A_{res} of the residual vibration (2) with respect to the eigenfrequency and damping, to zero (first-order sensitivity). The developed framework yields similar results by considering two systems in (20) with γ equal to zero and eigenfrequencies that are slightly different from the nominal value, e.g.:

$$\omega_n = 2\pi \pm 0.01\pi[rad/s]. \tag{25}$$

A traditional way for testing the robustness of the developed prefilter, is by plotting the vibration error, that is, the maximum amplitude of the residual vibration as a percentage of the amplitude of the desired point-to-point motion, versus the normalized natural eigenfrequency of the system, which is defined as the ratio of the natural eigenfrequency of the perturbed system ω_n and the nominal system $\omega_{n,0} = 2\pi$ rad/s [1]. This is shown in fig. 3 for both the ZVD approach (solid line) and for our robust approach (dashed line). The zoom shows the effect of the different approaches. The maximum difference between these approaches is $0.25\ 10^{-3}$ at the point where $\omega_n/\omega_{n,0} = 1$.

The sensitivity approach of the EI filter design is based on imposing zero residual vibration for two systems with eigenfrequencies that are significantly different from the nominal value, e.g.:

$$\omega_n = 2\pi \pm 0.1\pi[rad/s]. \tag{26}$$

This is often allowed as the nominal eigenfrequency is only known approximately and robustness is desired over a large interval. The developed framework reproduces this filter design exactly by considering two systems in (20) with the same eigenfrequencies as in (26) and γ equal to zero.

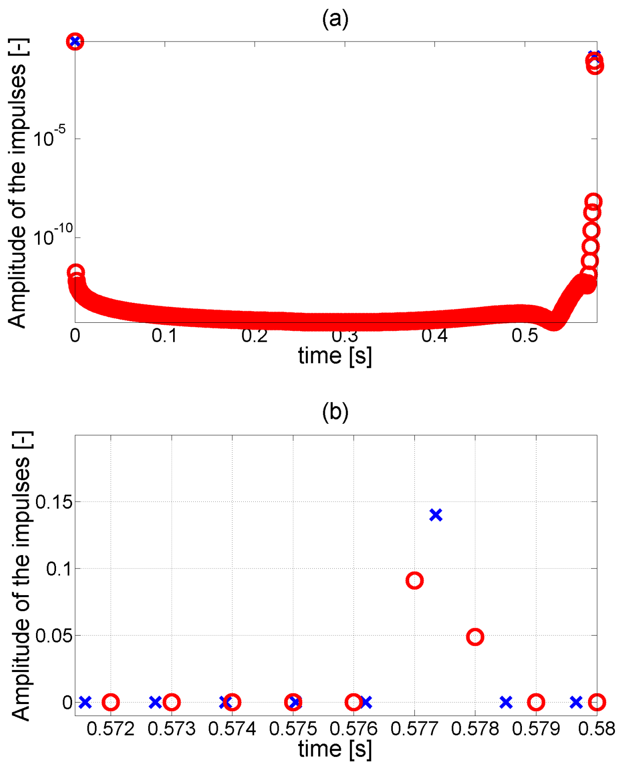

Figure 2: Impulses of a prefilter where the ideal prefilter is not possible due to the chosen discretization step T_s. The acquired solution (O) is very close to the ideal solution (X). (b) zooms linearly scaled in on (a) in the neighborhood of the second impulse.

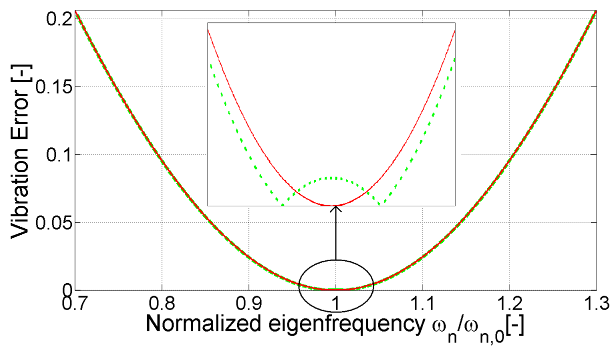

Figure 3: Vibration error versus system natural frequency: the ZVD result (solid line) and the result of the prefilter designed with the developed framework (dashed line).

4 Experimental validation

This section discusses the experimental validation of the developed framework on a two-DOF mass-spring-damper system. Fig. 4 shows a picture and a schematic drawing of the setup. The system is excited by a position controlled hydraulic piston with position $p(t)$. The system input is the reference signal for the piston position controller. The system output is the position of the upper mass $x_1(t)$. This is measured using a laser distance sensor.

This system is identified as a fifth-order discrete time state space model with a sample input $T_s = 0.001s$. This identification is based on frequency response function measurements that are obtained from a multisine

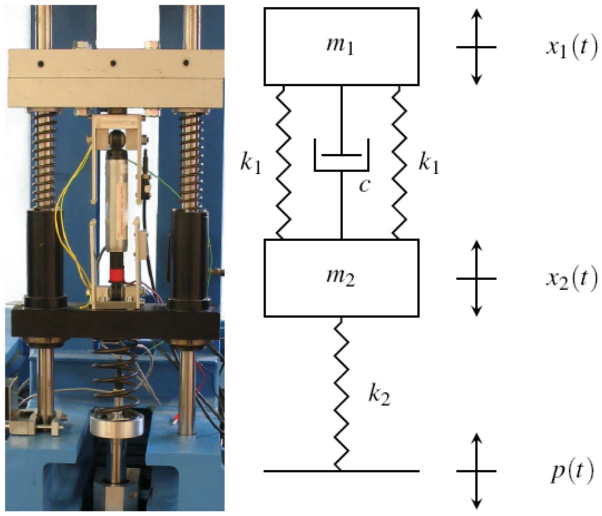

Figure 4: The controlled fifth order system.

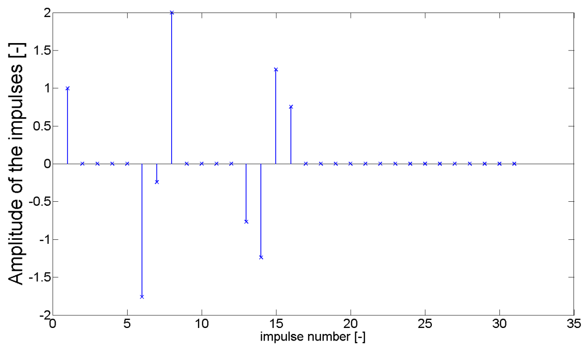

Figure 5: The resulting prefilter for the fifth order system. This prefilter is developed with an inputconstraint of $1cm$, and an outputconstraint of maximum 5% overshoot and no undershoot.

excitation with a frequency content between 0.1Hz and 10Hz [15]. For applying the developed framework, this model is transformed to discrete time with a sample period of $T_s = 0.01s$.

This model contains two pairs of complex conjugated poles originating from the two flexible modes of this system and one real pole introduced by the band limited piston position controller (Table 1).

Table 1: Poles of the considered fifth order system

frequency [rad/s]	damping [-]
$\omega_0 = 2.6205 \times 2\pi$	$\zeta_0 = 0.157\%$
$\omega_1 = 7.7926 \times 2\pi$	$\zeta_1 = 0.293\%$
1 real pole at 214	/

First, a nonrobust input shaping prefilter is designed for this system. This prefilter is developed with the following specifications: an input constraint of $|U| = 1cm$, a maximum overshoot of 5% of the upper mass displacement and no undershoot for the upper mass displacement. The lower and upper bounds for the bisection algorithm are $K_l = 0$ and $K_u = 200$, respectively. The upper bound corresponds to 5 times the period of the lowest eigenfrequency of the system, and is hence quite conservatively chosen. The resulting prefilter is shown in fig. 5.

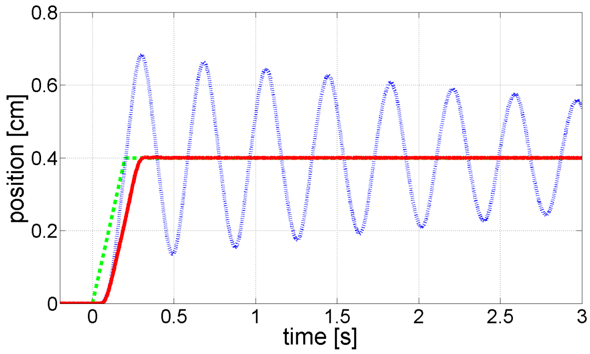

Figure 6: Validation of the nonrobust input shaping prefilter. The desired motion $r(t)$ (dashed line) is shown, and also the resulting system response with prefilter (solid line) and without prefilter (dotted line).

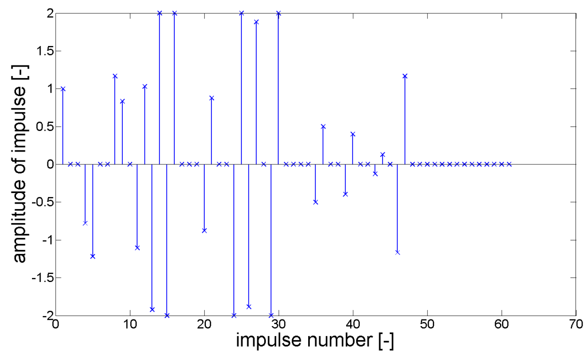

Figure 7: The resulting prefilter for the fifth order system. This prefilter is developed with an inputconstraint of $1cm$, an outputconstraint of maximum 5% overshoot and no undershoot. Also, robustness is imposed.

This prefilter is applied to a desired motion $r(t)$, which is a ramp with a duration of $0.2s$ and a displacement of $0.4cm$ (see fig. 6, dashed line). To avoid nonlinear effects in the system due to limited oil flow, a ramp motion instead of a step is used, in combination with the hard input constraint on u.

Fig. 6 shows the resulting behavior of the system $x_1(t)$ if the reference (dashed line) is applied to the prefilter. This behavior is shown without a prefilter (dotted line) and with a prefilter (solid line). This clearly shows that the developed prefilter converts the reference in a better suited input signal, although it also clearly evokes a move time penalty. This move time penalty is however smaller then if only positive impulses would be applied, namely $0.16s$ as compared with $T_0/2 = 0.1908s$ where T_0 is the period of the first eigenfrequency.

Next, a robust prefilter is designed. This prefilter not only makes the residual vibration zero for the nominal system, but also for systems that differ 10 % of the nominal system (for the first eigenfrequency). Considering the same constraints on input and output as before, this results in the filter shown in fig. 7. This filter contains much more impulses than the non robust prefilter. This is due to the combined constraints on robustness, input and output. It should be noted however that although the test setup is quite limited in terms of input due to the nonlinear oil flow, the system still behaves as expected and hence does not have problems to apply and follow the requested input.

Both this robust prefilter and the non robust are applied to a perturbed system. This perturbed system is the nominal system with masses added. Fig. 8 shows the resulting behavior, if a ramp input is applied as before (dashed line). The system has eigenfrequencies, which differ 20 % of the eigenfrequencies of the nominal system. The robust prefilter (solid line) yields much less residual vibrations than the non robust prefilter (dotted line), but introduces a bigger move time penalty.

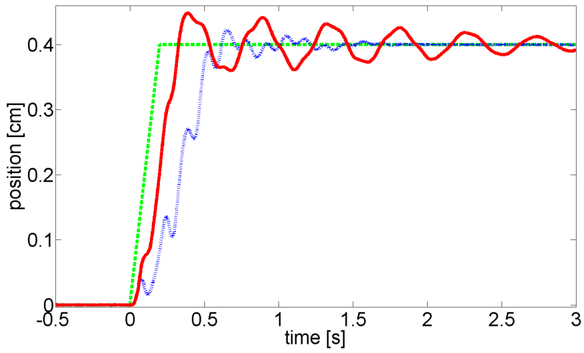

Figure 8: Comparison of a robust and non robust prefilter for a perturbed system. The desired motion $r(t)$ (dashed line) is shown. Also the resulting system response with a robust prefilter (dotted line) and with a non robust prefilter (solid line). There is gain of 50% in settling time due to the robustness.

5 Conclusion

This paper presents a new framework to design input shaping prefilters for flexible motion systems. Unlike the traditional approaches which are mainly analytic and derived for second order system, the presented approach is based on numerical optimisation of a linear problem. Moreover, the developed framework works irrespective of the order of the considered system. Also, constraints on input, output and robustness are naturally introduced. Numerical tests show that the developed approach is able to reproduce existing approaches for second order systems. Experiments on a 2DOF test setup prove, that also higher order systems can be efficiently handled. This experiments also show the requirement of robustness in the framework.

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