Filtered-X LMS vs repetitive control for active structural acoustic control of periodic disturbances

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Abstract

This paper presents a comparison of repetitive control and narrowband filtered-X LMS feedforward control, both applicable in the field of active structural acoustic control (ASAC) and active noise control (ANC) of periodic disturbances. This can be useful for example in rotating machinery, where the disturbance is often determined by the rotational speed. A frequency domain relation between the disturbance and the error signal is presented for the filtered-X LMS algorithm. This relation leads to a frequency domain convergence criterion which is consistent with existing literature and experimental observations. Furthermore, both control strategies are compared based on these frequency domain relations. The comparison shows that an inverse based repetitive controller behaves as an harmonic canceler, while the filtered-X LMS algorithm deviates from this behaviour at frequencies between the harmonics. Finally, the paper suggests the possibility to further match the behaviour of both algorithms, and to improve the performance for ASAC of periodic disturbances.

1 Introduction

This paper presents a comparison of two control strategies which can be applied in the field of active structural acoustic control (ASAC) and active noise control (ANC); repetitive control (RC) [1] and the filtered reference least mean squares algorithm (FXLMS) [2, 3]. Although repetitive control is not often associated with acoustic control, the RC algorithm has been successfully applied to the active control of rotating machinery [4], using a novel design strategy, suited for systems with a high modal density [5, 6]. In this kind of ASAC application, where the disturbance is periodic with a frequency determined by the rotational speed, the repetitive control strategy presents a valid alternative for the commonly used feedforward narrowband filtered-X LMS algorithm. Therefore, it is interesting to compare the performance and stability properties of both algorithms on a theoretical basis.

Section 2 presents a short theoretical overview of both repetitive control and filtered-X LMS. It presents the frequency domain relation between disturbance and error signal for repetitive control. Under certain assumptions, a similar relation for the narrowband filtered-X LMS algorithm is deduced. This relation differs from existing literature in that a model of the plant appears in the relation [3, 7]. Its correctness however, is successfully validated by simulations. Furthermore, it also serves as the basis for a new frequency domain convergence criterion, which is consistent with existing literature and experimental observations.

Based on the theoretical considerations, section 3 shows that the filtered-X LMS algorithm and repetitive control have a similar behaviour, i.e. as a harmonic canceler. At frequencies between the harmonics however, the filtered-X LMS algorithm deviates from the behaviour of the repetitive controller. Secondly, it is shown that also the stability criterion is similar in both algorithms. Furthermore, a visual representation of this criterion shows that the performance of the filtered-X LMS algorithm could be improved by a more appropriate choice of the control filters.

2 Theoretical background

This section gives a short introduction in repetitive control and the narrowband filtered-X LMS algorithm. First, a repetitive control scheme is presented, and the frequency domain relation between disturbance and error signal is given, as well as the stability criterion. For the filtered-X LMS algorithm, an equivalent relation in frequency domain is presented. Based on this relation, a convergence criterion for the filtered-X LMS algorithm can be deduced, which is consistent with existing literature.

2.1 Repetitive control

Figure 1 shows the repetitive control scheme used for ASAC, with k the discrete time instant. The input to the plant $P(z)$ is the voltage to the control actuator. Since in acoustic control the goal is to reduce the noise as much as possible, in the presence of the disturbance $d(k)$, the scheme lacks a desired reference signal, such that the output $y(k)$ is the error signal. The output $y(k)$ is therefore a measure related to the radiated noise. This can be an acceleration measurement on the controlled structure, or a direct measurement of the noise.

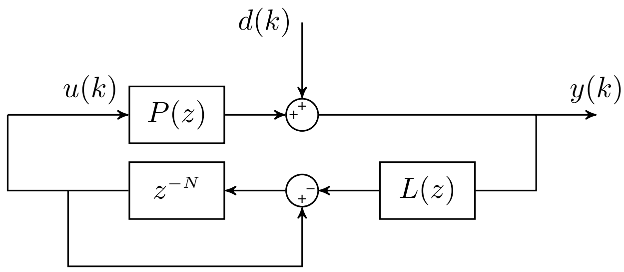

Figure 1: *Repetitive control scheme for ASAC applications.*

Although the disturbance $d(k)$ is arbitrary, the scheme is only effective at disturbances with a fundamental period of NT_s, with T_s the sampling period. This is due to the presence of the memory loop (feedback loop around z^{-N}) with a memory of N samples. Including the memory loop in the controller is based on the internal model principle, which states that in order to control a certain disturbance, the controller must contain a model of this disturbance. Since this memory loop is able to generate harmonic signals with a fundamental period of NT_s, this scheme cancels all these harmonic disturbances. By varying N, the scheme becomes adaptive and is able to track variations in the disturbing frequency. The filter $L(z)$ is the control filter which determines the performance of the control system and incorporates a cut-off to ensure stability at high frequencies.

Since the repetitive control strategy is a feedback strategy, the sufficient but not necessary condition for asymptotic stability is based on the small gain theorem (e.g. [8]), and states that $P(z)$ must be asymptotically stable and that

$$|1 - L(z)P(z)| < 1, \quad z = e^{\jmath\omega T_s}, \omega \in [0, \pi]. \tag{1}$$

For this criterion to be fulfilled, the filter $L(z)$ must be chosen such that the Nyquist plot of $L(z)P(z)$ lies within a unit circle around the point $(1, 0)$. The relation between the disturbance $d(k)$ and the remaining error $y(k)$ then becomes:

$$Y(z) = \frac{1 - z^{-N}}{1 - z^{-N}(1 - L(z)P(z))} D(z). \tag{2}$$

It is clear from this relation that for perfect cancelation of the harmonics of the fundamental frequency N/T_s, $L(z)$ must equal the inverse of $P(z)$.

2.2 Filtered-X LMS algorithm

When the disturbance is periodic and the radiated noise is dominated by the fundamental frequency and its harmonics, the narrowband version of the filtered-X LMS algorithm can be used [3]. The fundamental frequency must then be given by a reference signal, like a tacho signal, usually available in applications where the disturbance is periodic (e.g. rotating machinery). Figure 2 shows the resulting control scheme, where it is assumed that the disturbance is periodic with fundamental period NT_s. Like in the RC case, it will be shown that although the disturbance is not restricted to a periodic signal, the algorithm is only effective at harmonics of the fundamental frequency. This is contrary to the broadband version, which is effective over a broad frequency range. In that case however, it is imperative that there is a time delay between the reference signal $x(k)$ and the error signal $y(k)$, which usually limits the choice of $y(k)$ to an acoustic measurement. In the narrowband version, both acoustic and structural measurements can be used.

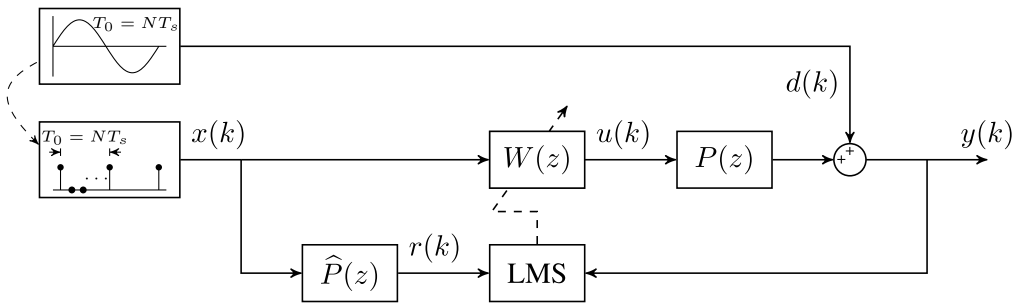

Figure 2: *Filtered reference LMS feedforward control scheme.*

In the following it is assumed that the reference signal $x(k)$ is a pulse train at the fundamental disturbance frequency N/T_s and that both the control filter $W(z)$ and the plant model $\widehat{P}(z)$ are modeled as FIR filters with length N. With these assumptions, it is possible to deduce a frequency domain relation between the disturbance $D(z)$ and the error signal $Y(z)$. In order to track variations in the disturbing frequency, the frequency of the pulse train needs to be adaptive, as well as the length of the filters $W(z)$ and $\widehat{P}(z)$.

The reference signal $x(k)$ is filtered by the control filter $W(z)$, which is updated according to following update law:

$$w_l(k+1) = w_l(k) - \mu r(k-l)y(k), \qquad l = 0, 1, \ldots, N-1, \tag{3}$$

with μ the update coefficient and $r(k)$ the reference signal, filtered by the plant model $\widehat{P}(z)$. According to [9], the filter coefficients converge to their optimal value as long as the phase of $P(z)$ and $\widehat{P}(z)$ do not differ by more than 90°. With $x(k)$ a pulse train, the reference signal $r(k)$ becomes:

$$\begin{aligned} r(k) &= \sum_{m=0}^{N-1} \hat{p}_m x(k-m) \\ &= \sum_{m=0}^{N-1} \hat{p}_m \sum_{n=-\infty}^{\infty} \delta(k-m-nN) \\ &= \hat{p}_{j(k)}, \end{aligned} \tag{4}$$

with $j(k) = k \bmod N$ (remainder on division of k by N). This modifies the update law (eq. 3) into:

$$w_l(k+1) = w_l(k) - \mu \hat{p}_{j(k-l)} y(k), \qquad l = 0, 1, \ldots, N-1, \tag{5}$$

To come to an analogous relation to eq. 2 between the disturbance and the remaining error, the control signal $u(k)$ is written as follows:

$$\begin{aligned} u(k) &= \sum_{l=0}^{N-1} w_l(k)x(k-l) \\ &= \sum_{l=0}^{N-1} w_l(k) \sum_{n=-\infty}^{\infty} \delta(k-l-nN) \\ &= w_{j(k)}(k). \end{aligned} \tag{6}$$

If the update starts with all filter coefficients equal to zero, it is possible to write the value of coefficient w_l at time k as follows:

$$w_l(k) = -\mu \sum_{i=0}^{k-1} \hat{p}_{j(i-l)} y(i), \tag{7}$$

resulting in following expression for the filter output $u(k)$:

$$\begin{aligned} u(k) &= w_{j(k)}(k) \\ &= -\mu \sum_{i=0}^{k-1} \hat{p}_{j(i-j(k))} y(i) \\ &= -\mu \sum_{i=0}^{k-N-1} \hat{p}_{j(i-j(k))} y(i) - \mu \sum_{i=k-N}^{k-1} \hat{p}_{j(i-j(k))} y(i) \\ &= u(k-N) - \mu \sum_{i=0}^{N-1} \hat{p}_i y(k-N+i). \end{aligned} \tag{8}$$

Taking the z-transform yields:

$$\begin{aligned} U(z) &= z^{-N}U(z) - \mu \sum_{i=0}^{N-1} \hat{p}_i z^{-N+i} Y(z) \\ U(z) &= z^{-N}U(z) - \mu z^{-N} \widehat{P}(z^{-1}) Y(z) \\ U(z) &= -\mu \frac{z^{-N}}{1-z^{-N}} \widehat{P}(z^{-1}) Y(z). \end{aligned} \tag{9}$$

With $Y(z) = U(z)P(z) + D(z)$ (see figure 2), this leads to following relation between disturbance and error signal:

$$Y(z) = \frac{1-z^{-N}}{1-z^{-N}(1-\mu\widehat{P}(z^{-1})P(z))} D(z). \tag{10}$$

This relation is valid for the Filtered-X LMS scheme from figure 2, with $\widehat{P}(z)$ and $W(z)$ FIR filters of length N, depending on the fundamental frequency. Although in normal operation the disturbance is periodic, this is not taken into account in the derivation of eq. 10, such that the relation is also valid for a random disturbance, as long as N is chosen to be a fixed number of sample periods.

Relation 10 differs from what can be found in literature [3, 7], where the plant model $\widehat{P}(z^{-1})$ does not appear in the relation. As a validation of the correctness of relation 10, a simulation of the Filtered-X LMS algorithm is performed on a simple plant, of which the transfer function $P(z)$ is shown in figure 3. The discrete model $\widehat{P}(z)$ of the plant $P(z)$ is a FIR filter with length 50, at a sampling rate of 5 kHz. Like the plant model, the control filter $W(z)$ is implemented as a FIR-filter of length $N = 50$, with all coefficients initially equal to zero. In order to assess the behaviour of the system on all frequencies up to the Nyquist frequency, the disturbance is a random signal. The reference signal $x(k)$ is a pulse train with a period of 50 samples. The resulting control scheme is shown in figure 4 and the relation between the disturbance $d(k)$ and the error

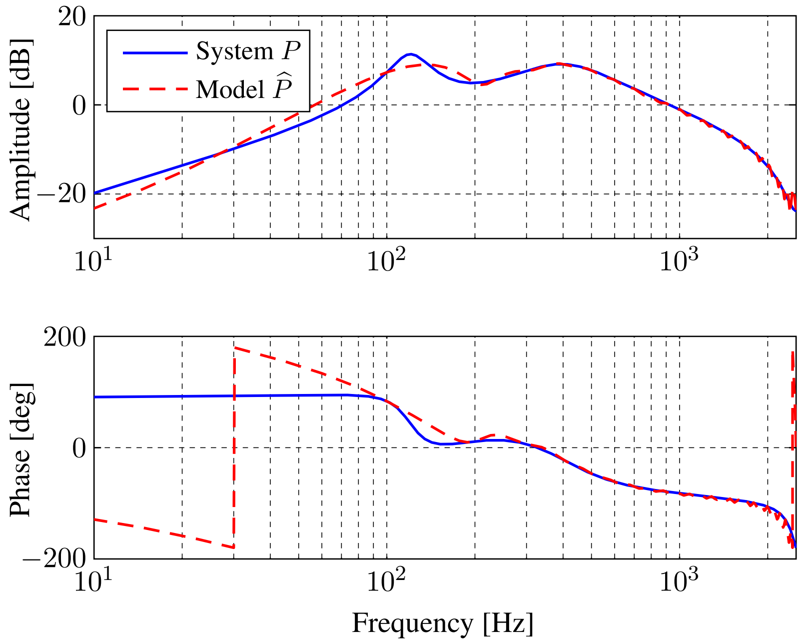

Figure 3: *System and system model used for simulation of the Filtered-X LMS algorithm.*

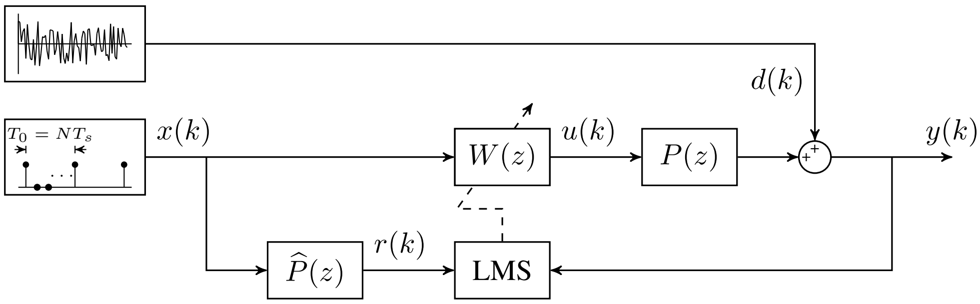

Figure 4: *Filtered reference LMS feedforward control scheme with random disturbance.*

signal $y(k)$, simulated and calculated from relation 10, is given in figure 5, with the update coefficient μ chosen equal to 0.15.

Within the aforementioned assumptions, two major observations are clear from the simulation. Firstly, the simulation shows that relation 10, presented in this paper, correctly predicts the behaviour of the Filtered-X LMS algorithm. Secondly, the algorithm behaves as a harmonic canceler, effectively reducing the fundamental frequency of 100 Hz ($N = 50$ at 5 kHz) and all its harmonics. At intermediate frequencies, the algorithm has a negative effect on the error, resulting in an increase of the error signal.

Relation 10 does not only predict the behaviour of the narrowband Filtered-X LMS algorithm, but can also be used to analyse the convergence of the system. For stability of the transfer function between $D(z)$ and $Y(z)$, the roots of its denominator must lie within the unit circle. This criterion is fulfilled if:

$$|1 - \mu\widehat{P}(z^{-1})P(z)| < 1, \quad z = e^{\jmath\omega T_s}, \omega \in [0, \pi]. \tag{11}$$

This paper will not go into further detail on this criterion, but since the phase of $\widehat{P}(z^{-1})$ is opposite to that of $\widehat{P}(z)$, this confirms that the algorithm converges when $\widehat{P}(z)$ and $P(z)$ do not differ more than 90° and when the update coefficient μ is chosen appropriately [9].

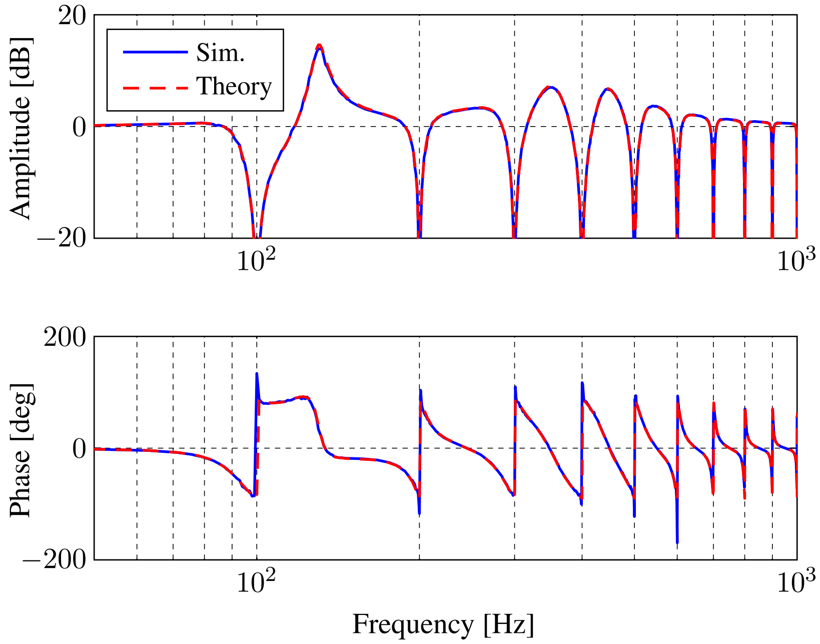

Figure 5: *Relation between disturbance $D(z)$ and error signal Y(z) in simulation and theoretically, from eq. 10.*

	Repetitive control	***Filtered-X LMS***
Performance	$\frac{Y(z)}{D(z)} = \frac{1-z^{-N}}{1-z^{-N}(1-L(z)P(z))}$	$\frac{Y(z)}{D(z)} = \frac{1-z^{-N}}{1-z^{-N}(1-\mu\widehat{P}(z^{-1})P(z))}$
Stability	$\lvert 1-L(z)P(z)\rvert < 1$	$\lvert 1-\mu\widehat{P}(z^{-1})P(z)\rvert < 1$

Table 1: *Performance and stability properties of repetitive control and filtered-X LMS.*

3 Comparison of Filtered-X LMS and Repetitive control

Based on the relation between disturbance $D(z)$ and error signal $Y(z)$ for both repetitive control and narrowband filtered-X LMS, this section compares both control strategies for use in active structural acoustic control applications. Stability and performance properties are investigated and simulations are presented.

3.1 Performance

The performance of an acoustic control system is determined by the achievable reduction of the disturbance signal and, in case of non-stationary signals, the rate of convergence. The achievable reduction is given for RC by equation 2 and for FXLMS by equation 10, repeated in table 1. The equations relate the disturbance $D(z)$ to the error signal $Y(z)$. From the comparison, it is obvious that both algorithms have a similar behaviour, i.e. that of a harmonic canceler. The performance in the frequencies between the harmonics, is determined respectively by the filter $L(z)$ and $\mu\widehat{P}(z)$ for RC and FXLMS.

Figure 6 shows the performance of the RC and FXLMS algorithm for the plant $P(z)$ of figure 3, when the disturbance is a white noise signal, and with a fundamental frequency of 100 Hz ($N = 50$ at 5 kHz). For the RC case, $L(z)$ is an exact inverse of the plant, such that the algorithm behaves as an ideal harmonic canceler. The maximum amplification of the intermediate frequencies is 6 dB. For the FXLMS case, $\widehat{P}(z)$ is a FIR-

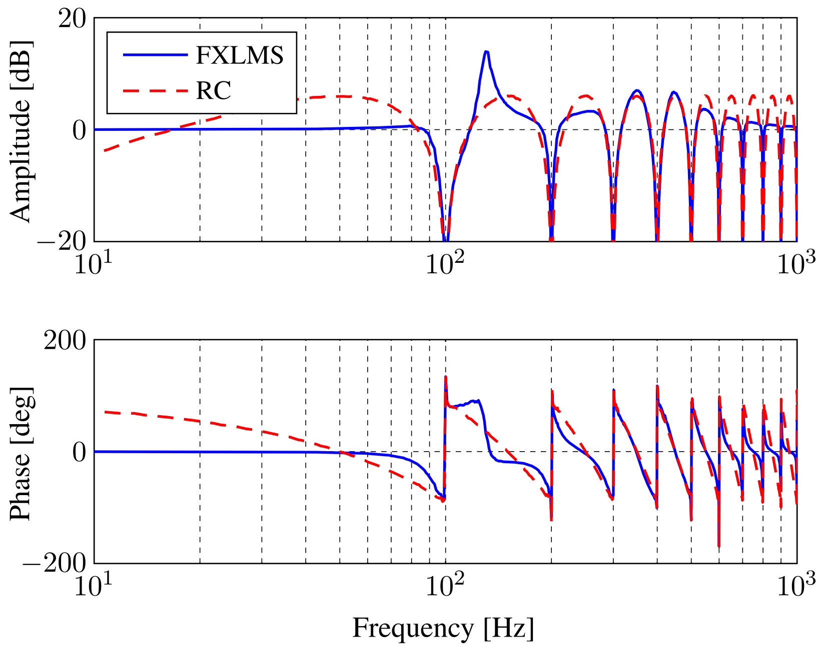

***Figure 6:** Comparison between the performance of a Filtered-X LMS and a repetitive controller.*

model of the plant $P(z)$, with 50 elements, as shown in figure 3. The convergence coefficient μ is equal to 0.15. Since μ is not frequency dependent, $\mu\widehat{P}(z^{-1})P(z)$ is not equal to 1 over the whole frequency range like $L(z)P(z)$ in the RC case, and the FXLMS algorithm does not behave as an ideal harmonic canceler. In the frequencies between the harmonics, the reduction or amplification is determined by the amplitude of $\mu\widehat{P}(z^{-1})P(z)$. At high frequencies, where the plant usually exhibits a roll-off, this has the advantage that the intermediate amplification decreases, compared to the RC-case. At resonance frequencies of the plant on the other hand (e.g. at 120 Hz), the amplification is higher than in the RC-case.

3.2 Stability

Based on the frequency domain representation, the stability criterion (usually called convergence criterion in the feedforward framework) for the filtered-X LMS algorithm is given by eq. 11, compared to the criterion for RC in table 1. As in the performance relation, there is an important analogy between both strategies; in RC, the phase of $L(z)$ needs to oppose the phase of $P(z)$ within a margin of 90°. In FXLMS, the phase of $\widehat{P}(z)$ needs to equal the phase of $P(z)$ within a margin of 90°.

Moreover, just like for learning controllers like repetitive and iterative learning control [10], the stability criterion has a visual interpretation; for the algorithm to converge, the Nyquist plot of $\mu\widehat{P}(z^{-1})P(z)$ must lie within the unit circle around the point (1,0). Figure 7 shows that for the example used in this paper, the criterion is fulfilled. In case of a perfect plant model $\widehat{P}(z)$ without any phase mismatch, the Nyquist plot of $\mu\widehat{P}(z^{-1})P(z)$ lies on the real axis. For RC, where $L(z)$ is an exact inverse of the plant $P(z)$, the Nyquist plot of $L(z)P(z)$ reduces to the point (1,0).

The observation that the Nyquist plot of $L(z)P(z)$ of an ideal inverse-based repetitive controller reduces to a single dot, shows that there is still a large margin in the design of $L(z)$. In a lot of cases, an exact plant inverse is not feasible due to the complex nature of the controlled plant. Moreover, this "ideal" behaviour is not always required over the whole frequency range up to the Nyquist frequency. Therefore, [5, 6] presents a novel design approach taking advantage of the remaining stability margin, and especially suited for systems with a high modal density.

A second consideration relates to the Nyquist plot of $\mu\widehat{P}(z^{-1})P(z)$, which lies completely on the real axis in case of a perfect plant model $\widehat{P}(z)$. In order to obtain the same behaviour as the RC algorithm, a proper

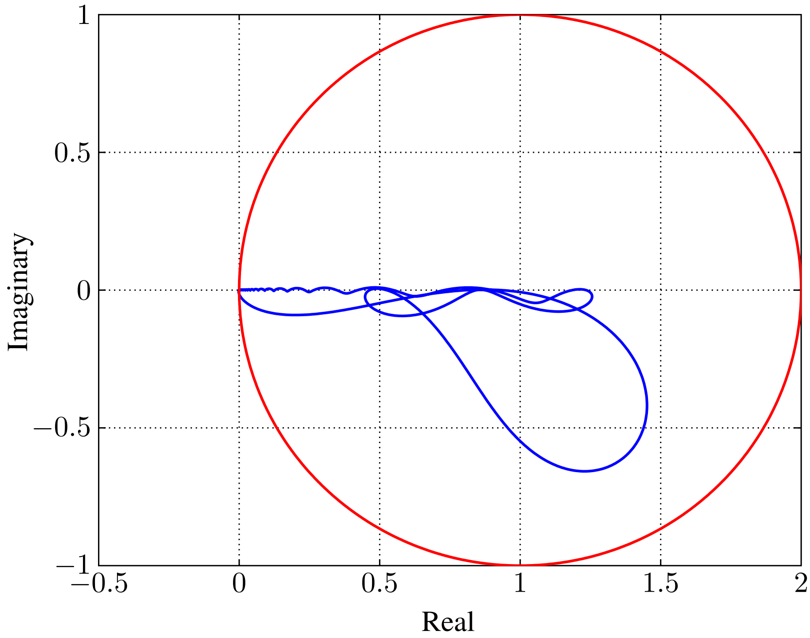

Figure 7: *Visual representation of the filtered-X LMS convergence criterion (eq. 11) with $\mu\widehat{P}(z^{-1})P(z)$ (blue) contained in the convergence circle (red).*

selection of $\mu\widehat{P}(z)$ must reduce this line to the point (1,0). On the other hand, like in the RC-case, this behaviour might not always be required. This paper will not go into further detail on the design of a filtered-X LMS controller, but it is clear that (i) a frequency dependent convergence coefficient μ and (ii) a more appropriate choice of $\widehat{P}(z)$ can further improve the performance of the algorithm.

4 Conclusions

This paper presents a comparison of repetitive control (RC) and narrowband filtered-X LMS (FXLMS), both applicable in the field of active structural acoustic control (ASAC) and active noise control (ANC) of periodic disturbances. A typical application is for example rotating machinery, where the disturbance is often determined by the rotational speed. A control strategy like repetitive control or filtered-X LMS feedforward control reduces the fundamental disturbance frequency and its harmonics, which effectively reduces the resulting noise radiation.

A first part of the paper presents a brief introduction in repetitive control and filtered-X LMS. For FXLMS, a frequency domain relation between the disturbance and the error signal is presented. Based on this relation, a frequency domain convergence criterion can be deduced, consistent with existing literature and experimental observations.

In a second part, both control strategies are compared with respect to performance and stability. The performance of an ideal inverse-based RC controller is that of a harmonic canceler. The FXLMS algorithm exhibits the same reduction at harmonic frequencies, but differs at intermediate frequencies. Taking into account the stability criterion which is also similar for RC and FXLMS, it is suggested that the performance of the FXLMS algorithm can be further improved by a more appropriate choice of the filter and the convergence coefficient.

Finally, for practical ASAC and ANC applications, it is shown that not only FXLMS, but also RC is a valuable alternative, although repetitive control is not often associated with acoustic control. The goal of this paper however is not to suggest on of both strategies, but to present a first comparison. Nonetheless it is clear that due to the assumptions regarding the control filters in the presented FXLMS strategy, the RC strategy leaves more design freedom. Since it is however believed that these assumptions will not influence the practical use of the FXLMS strategy a lot, further investigations will focus on the choice of these filters as well as on the practical implementation.

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