

# Tracking Performances of Cascade and Sliding Mode Controllers with Application to a XY Milling Table

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## Abstract

Design and implementation of robust tracking controllers for linear feed drive high speed machine tools are the primary objectives of this work. Cutting forces and friction forces limit tracking accuracy and their influences are minimised through robust controller design. This paper discusses the design and tracking performances of a traditional cascade and a sliding mode controller for a X-Y milling table. Circular tests are performed at selected tracking speeds and circle radii for tracking performance analysis. The tracking error for cascade P/PI controller is proportional to the reference tracking speed and is inversely proportional to the velocity gain value of the position loop controller. Speed and acceleration feedforward further reduce this tracking error. Quadrant glitches, a product of nonlinear friction at the points of velocity reversal, are observed. Sliding mode controller exhibits better tracking accuracy and dynamic stiffness and yields a significant reduction of the quadrant glitches.

## 1 Introduction

Most electromechanical feed drives of table positioning systems rely on the ball screw mechanism for the rotational to linear motion conversion. According to Pritschow [1], the presence of a lead screw in the transmission mechanism gives rise to several performance limitations, namely: (1) backlash, (2) transmission errors from pitch tolerances of the leadscrew and (3) compliance yield a low first mechanical natural frequency of the system, resulting in smaller system's bandwidth.

On the other hand, direct linear drives, in the absent of any mechanical transmission element, eliminate the limitations associated with the ball screw drives. Figure 1 illustrates the opposing mechanical structure of the two drives systems with application to positioning control.

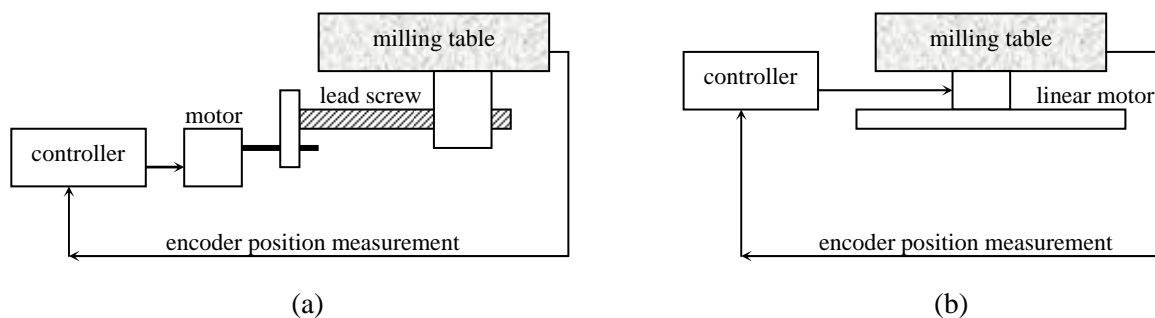


Figure 1: (a) ball screw feed drive and (b) linear drive

Linear drives have opened the window for the application of high speed positioning system with higher accuracy, velocity and acceleration than those possible with conventional systems. High-speed machining is attractive in a sense that it creates potential for higher productivity due to shorter machining time. However, high-speed machining creates another challenge, that is, the machining process requires the feed drives to be robust against fast acting disturbances. This is even more important with linear drives since disturbances are directly acting on the drives as clearly indicated in Figure 1(b).

Motion tracking controllers are designed with the fundamental objective of achieving highest possible position tracking accuracy. This, however, is significantly influenced by friction forces acting closely at the load and cutting forces from the machining process. A prominent tracking controller that exists in majority of servo motion control systems is the classical cascade controller. Various improvements and modifications to this controller have been suggested in the literature with the aim of improving the tracking performance. For example, Doenitz [2] has analysed the tracking error of combined cascade controller with a proposed disturbance observer. Pritschow [3] on the other hand, presented different positioning behaviours between classical P/PI cascade controller and reduced state space controller. In addition to classical cascade controller, sliding mode controller, a well known nonlinear controller, is highly regarded for its disturbance rejection properties. Altintas [4] has demonstrated the contouring performance superiority of this controller against pole placement controller with feedforward friction and servo dynamics compensation. This paper presented the study on design, implementation, and tracking performance comparison between traditional cascade P/PI controller and the nonlinear sliding mode controller with application to linear feed drive high speed machine tools of an XY milling table. Tracking performance are compared by means of circular tests that were performed at different tracking speeds and circle radii. Comparisons are based on axial tracking errors, contour tracking errors, dynamic stiffness and quadrant glitches.

## 2 Basic Design of Tracking Controller

A well designed motion tracking controller reacts to the input reference trajectory signal with minimum tracking error and exhibits robustness against system uncertainties and disturbances. The following sections discuss designs of two tracking controllers, namely, classical cascade controller and sliding mode controller.

### 2.1 Cascade Controller

Classical cascade controller (see figure 2) consists of two distinct loops; an inner speed loop and an outer position loop. The speed filter transfer function generates the speed signal while applying low pass filtering of the excited measurement noise.

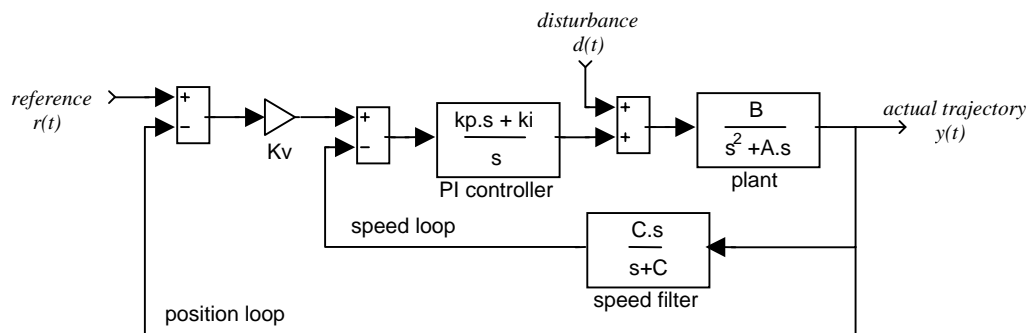
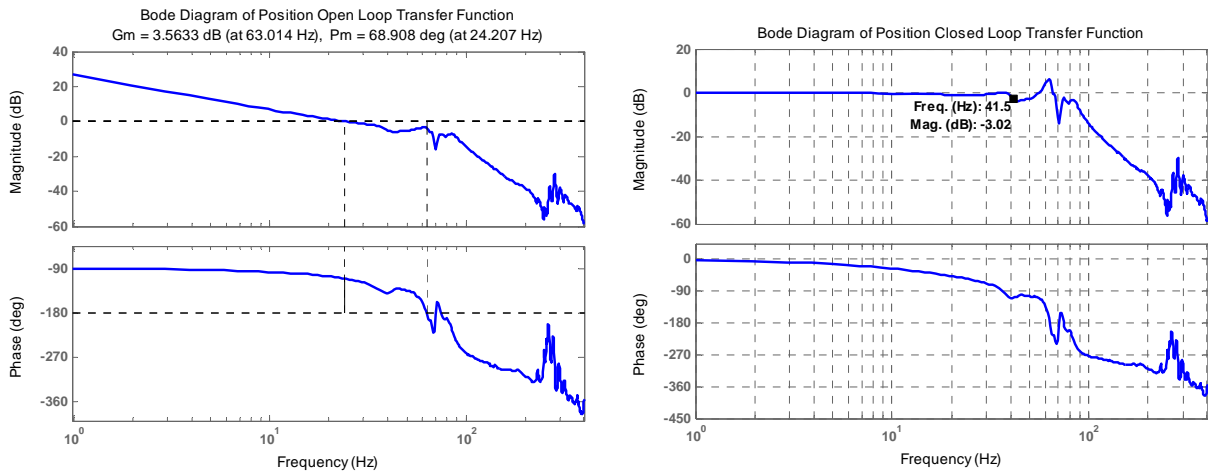


Figure 2: Schematic diagram of P/PI cascade controller

The closed-loop bandwidth of the inner speed loop is higher than the bandwidth of the outer position loop and this ensures that the dynamics delay of the inner loop is ignored by the outer position loop [5]. The controller is designed based on traditional open-loop shaping and is constructed and tuned from the speed loop to the position loop. A proportional (P) plus integral (I) control forms the speed loop. The controller parameters are designed from the knowledge of gain margin and phase margin of the resulting speed open-loop transfer function that is based on actual measurement of open-loop frequency response function of the system. The phase margin and the gain margin indicate the system stability margin and its transient response [6]. A proportional controller completes the position loop and hence the cascade controller scheme. Similar in the design of the speed loop, this proportional gain is selected based on the gain margin and the phase margin of the position open loop transfer function that incorporates the speed closed loop (see figure 3). The stability of each loop is confirmed by the Nyquist plot of the open loop transfer function.



**Figure 3: Bode diagrams of position open loop and closed loop transfer functions (y-axis)**

The following transfer functions relate input disturbance,  $d(s)$ , to output position,  $y(s)$ , and reference signal,  $r(s)$ , to the tracking error,  $e(s)$ :

$$\frac{d(s)}{y(s)} = \frac{mt_i s^3 + k_f k_p t_i s^2 + (1 + k_v t_i) k_f k_p s + k_f k_p k_v}{t_i s} \quad (1)$$

$$\frac{e(s)}{r(s)} = \frac{ms^2 + k_f k_p s}{ms^2 + k_f k_p s + k_f k_p k_v} \quad (2)$$

where;  $k_f$  is the motor constant,  $k_v, k_p$ , and  $t_i$  ( $t_i = k_p / k_i$ ) are the controller parameters, and  $m$  is the mass of the system. The transfer function given in equation (1) defines the dynamic stiffness of a system. It indicates the controller disturbance rejection capacity. At steady-state, for a constant tracking velocity, the tracking error is proportional to the tracking velocity,  $v$ , but is inversely proportional to the velocity gain factor,  $k_v$ .

$$e(\infty) = \frac{v}{k_v} \quad (3)$$

## 2.2 Sliding Mode Controller

Sliding mode control, a form of nonlinear control, is widely known for its low sensitivity to plant parameters variation and disturbances. It is also a form of variable structure control (VSC). According to Utkin [7], a variable structure system is a set of continuous subsystems with a proper switching logic. Hence, the control actions are discontinuous functions of disturbances, states, and the reference input. There are two design components of sliding mode control; namely, switching function and control law.

### 2.2.1 Switching Function

Switching function determines the dynamics transient response of a system. For a second order system, the sliding surface,  $s$ , is a function of the tracking error,  $e(t)$  and its time derivative:

$$s(e, \dot{e}) = \left( \frac{d}{dt} + \lambda \right)^{n-1} e; \quad e(t) = y(t) - r(t) \quad (4)$$

For  $n = 2$  (second order system),

$$s(e, \dot{e}) = \dot{e} + \lambda e \quad (5)$$

where,  $y(t)$  is the actual position,  $r(t)$  the desired position, and  $\lambda$  a positive constant. The choice of  $\lambda$  determines the transient response characteristics of the system.

### 2.2.2 Control Laws

Control law, on the other hand, consists of a signum function and the equivalent control which includes speed and acceleration feedforward:

$$u(t) = u_{equivalent} - K \cdot \text{sign}(s) \quad (6)$$

At the time of sliding, the switching function is equivalent to zero and hence;

$$s(e, \dot{e}) = 0 \quad (7)$$

Therefore, the dynamics during sliding is;

$$\dot{s}(e, \dot{e}) = \ddot{e} + \lambda \dot{e} = 0 \quad (8)$$

with;

$$\begin{aligned} \dot{e} &= \dot{y} - \dot{r} \\ \ddot{e} &= \ddot{y} - \ddot{r} \end{aligned} \quad (9)$$

Hence, for a simple mass-spring-damper system,

$$m \ddot{y} + c \dot{y} + k y = u \quad (10)$$

ignoring the disturbance function,  $d(t)$ , and by combining equations (8), (9) and (10), the control input becomes:

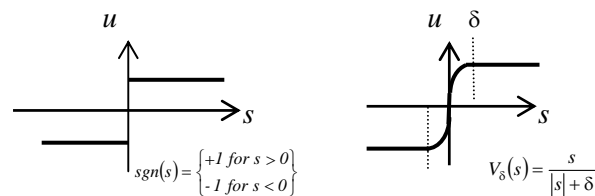
$$u_{equivalent} = m(\ddot{r} - \lambda \dot{e}) + c \dot{y} + k y \quad (11)$$

Equivalent control maintains the sliding motion. The discontinuous signum function with proportional gain  $K$  ensures that the states are attracted to and remain in the sliding surface in finite time. The discontinuous signum function, however, leads to high frequency switching. Instead of remaining on the sliding surface, the states are oscillating about the surface. This is widely known as chattering [8] and is a major disadvantage to sliding mode control application.

Several techniques are suggested in reducing chattering [9]. A continuous approximation of the discontinuous signum function is among the options. A sigmoid-like function of the following form replaces the discontinuous signum function [10];

$$V_{\delta}(s) = \frac{s}{|s| + \delta} \quad (12)$$

The positive constant,  $\delta$ , illustrates the degree of the continuous approximation. A comparison between the discontinuous signum function and its continuous approximation is shown in the following figure:



**Figure 4: Continuous smooth approximation of the signum function**

Instead of having an ideal sliding motion, the states lie within a specified boundary of the sliding surface, resulting in non-ideal sliding motion, known as pseudo-sliding.

### 3 Experimental Setup

The controllers' performance were validated on a xy feed table of a high-speed milling machine (see figure 5). The upper top stage (y-axis) is driven by a single ETEL linear motor while the bottom stage, which is the x-axis, is driven by two ETEL parallel linear motors. Two Heidenhain linear encoders provide the table two axes relative position measurements.



**Figure 5: A xy feed table with three linear drives for high speed milling application**

#### 3.1 Experimental Design Methodology

In the absence of a speed sensor, the speed signal is generated from the derivative of the encoder position measurements. This however, amplified the measurement noise. Thus, the derivative is combined with a first-order low-pass filter. The controllers of both axes were implemented on a dSPACE 1102 DSP controller board linking the host computer to the three ETEL drives.

Three different controllers were implemented: **(1)** a traditional cascade P/PI controller, **(2)** a cascade P/PI controller with speed and acceleration feedforward, and **(3)** a sliding mode controller. Circular test were performed to validate and compare tracking performance of these controllers.

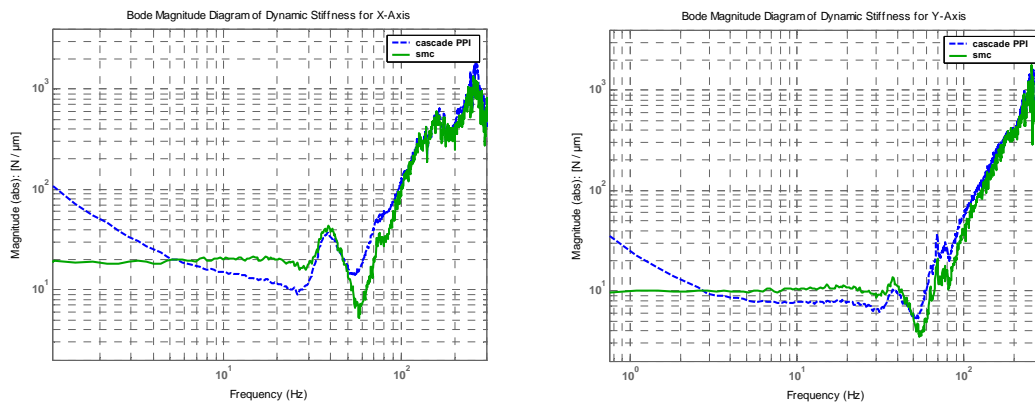
The controllers were designed to track circles of 15mm and 30mm radii, each at constant tracking speed of 0.001m/s, 0.005m/s, and 0.01m/s. Each axis position tracking error was recorded. The actual trajectory was compared to the reference trajectory. The system was not subjected to any external disturbance signal. In addition to tracking, dynamic stiffness of the cascade and sliding mode controllers was also investigated. In the absence of a reference signal, a band-limited white noise was introduced into the system as an input disturbance signal and the resulting position error was recorded.

## 4 Experimental Results

Circular tests provide clear indication of each controller tracking performance while dynamics stiffness measures the disturbance rejection property of the controllers.

### 4.1 Dynamic Stiffness

Dynamic stiffness is defined as the transfer function from the output position measurement to disturbance force (e.g. cutting forces). In a high-speed milling application, high machine dynamic stiffness is desired. It is important for the milling table to experience minimum position tracking error while subjected to disturbance signal such as the cutting force. The dynamic stiffness obtained with the P/PI cascade controller and the sliding mode controller for the x and the y axes are presented in figure 6. These were measured using band-limited white noise signal, acting as the disturbance force, while recording the position tracking errors.



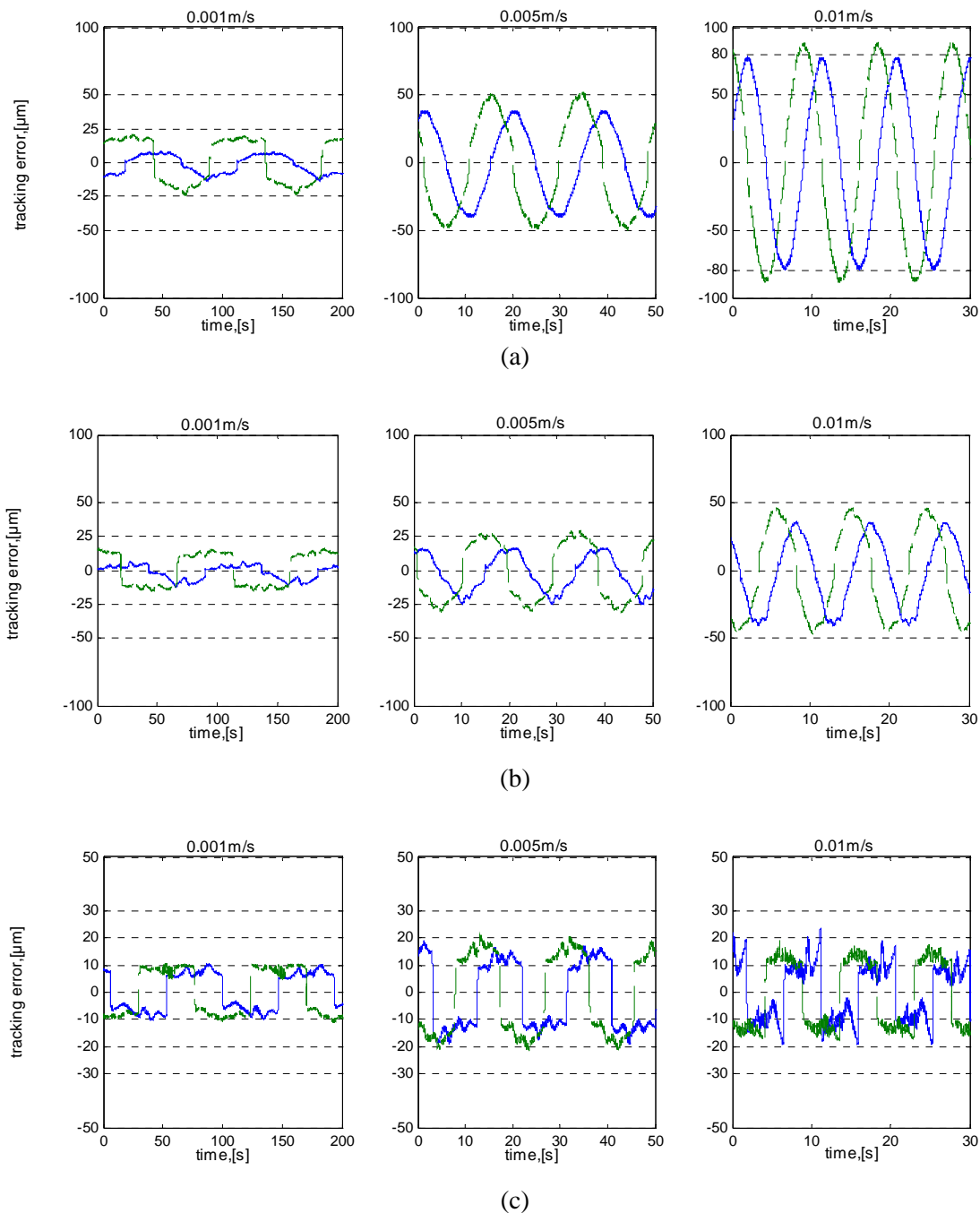
**Figure 6: Dynamics stiffness comparison of cascade and sliding mode controllers**

Figure 6 indicates that for both axes, sliding mode controller provides better stiffness than cascade controller within the bandwidth of the system, which is 40Hz. However, cascade controller displays higher stiffness (until about 4Hz) at much lower frequency range. This is due to the integrator within the cascade's speed loop controller. The results show that sliding mode has better disturbance rejection capacity than cascade controller from the frequency range of 4 Hz until the bandwidth of the system.

### 4.2 Tracking Error

Tracking error is defined as the difference between the reference trajectory and the system actual position. The tracking accuracy of a motion controller can be analysed by looking at the axial tracking error or of the contour tracking error. The axial tracking error indicates the individual axis tracking performance while contour tracking error describes both the x and the y axis tracking performance.

The following figures illustrate the axial tracking error of the controllers for circular tests with a radius of 15mm, executed at different tracking speeds.



**Figure 7: Axial tracking error with (a) cascade, (b) cascade with speed and acceleration feedforward, and (c) sliding mode for a 15mm radius circle tracking test at different speeds**

The results depicted above clearly shows that the tracking error increases with tracking velocity. The cascade controller yields the largest error for all tracking speeds. Speed and acceleration feedforward reduce significantly these errors. For example, at 0.005m/s and 0.01m/s, feedforwards reduce the tracking error with nearly a factor of two. Sliding mode yields the smallest tracking errors. The tracking errors are nearly half of that of the cascade controller with the speed and the acceleration feedforward. Similar observations and conclusions are made for circular tests with other radii. The tracking error results obtained for a circle with a 30mm radius are summarised in Table 1.

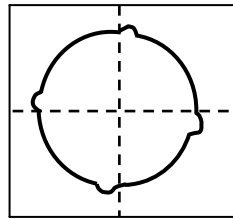
Tracking Speed / Controller	0.001m/s	0.005m/s	0.01m/s
Cascade	15 $\mu$ m	45 $\mu$ m	80 $\mu$ m
cascade + speed + acceleration FF	15 $\mu$ m	25 $\mu$ m	40 $\mu$ m
sliding mode	10 $\mu$ m	15 $\mu$ m	20 $\mu$ m

**Table 1: Maximum tracking error for 30mm radius circle tracking test at different tracking speeds**

Table 1, in comparison to figure 7, shows that tracking errors are not function of the circle dimension. Tracking error increases with increases in tangential velocity of the reference trajectory.

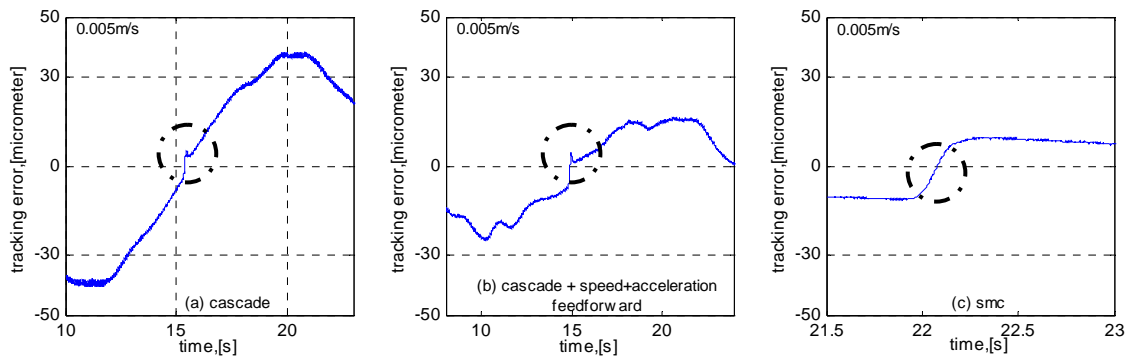
### 4.3 Quadrant Glitch

Quadrant glitch is a special form of contour tracking error. It is characterised by the presence of “spikes” in each quadrant of a circle, as illustrated in figure 8. This phenomenon arises from complex non-linear friction that occurs at the point of motor reversal motion.



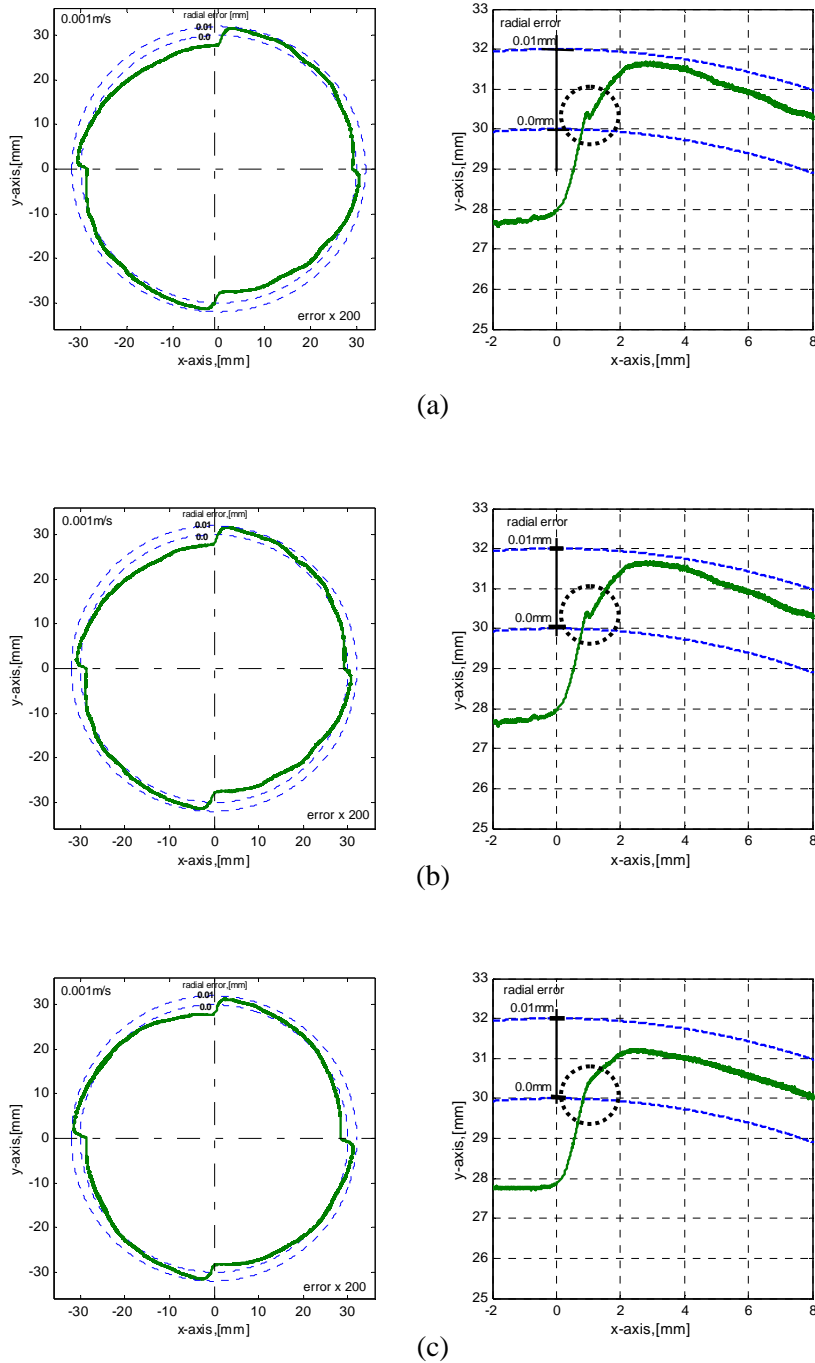
**Figure 8: Quadrant glitches as seen from circle contour test**

Sliding mode control is widely known for its robustness properties against matched uncertainties and disturbance [11]. In this analysis, the disturbance was in the form of friction forces alone. Quadrant glitches can be clearly recognised from the axial tracking error measurements. Figure 9 compares the axial tracking error of cascade controller, cascade with speed and acceleration feedforward, and sliding mode.



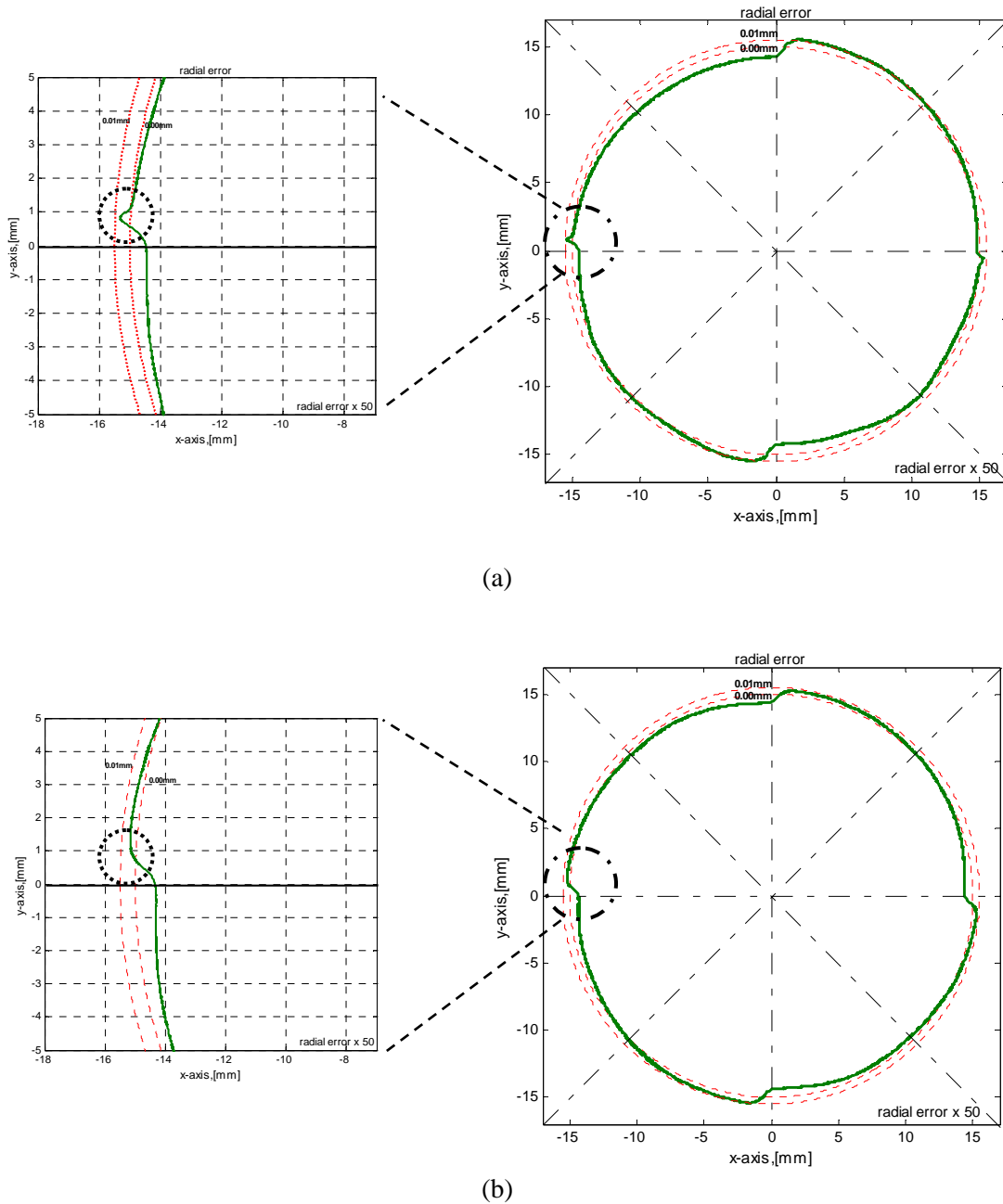
**Figure 9: Spikes as seen from axial tracking error for (a) cascade, (b) cascade with speed and acceleration feedforward, and (c) sliding mode controller**

Spikes are clearly visible from the axial tracking errors of both the cascade controllers. They are significantly reduced with the sliding mode controller. The axial tracking errors clearly indicate the sliding mode advantages in disturbance rejection of friction forces. Similar observation and conclusion can be derived from the contours of the circular tests shown in figure 10. The actual and the reference contours generated from the cascade controllers and the sliding mode controller, for a circle test of radius 30mm at a tracking speed of 0.001m/s, are compared. The actual contours were reconstructed with amplified radial error (200 times) for graphical presentation purposes. Spikes are clearly visible with both the cascade controllers.



**Figure 10: Quadrant glitch from circle test of radius 30mm and tracking speed of 0.001m/s for (a) cascade, (b) cascade with speed and acceleration feedforward, and (c) sliding mode control**

Similar comparison can be made for circular test of radius 15mm at a tracking speed of 0.005m/s. The results are shown below:



**Figure 11: Quadrant glitch from circle test of radius 15mm and tracking speed of 0.005m/s for (a) cascade with speed and acceleration feedforward, and (b) sliding mode control**

As seen from previous circular test, spikes are again clearly visible with the cascade controller. Figure 11 displays significant removal of the quadrant glitches with sliding mode controller. The radial errors at the quadrant positions for both cascade controllers and the sliding mode controller are within a range of +/- 10 micrometer.

## 5 Conclusions

Sliding mode controller exhibits higher dynamic stiffness and better tracking accuracy than the two classical cascade controllers. Speed and acceleration feedforward improve tracking accuracy of classical cascade controller. The classical cascade controllers clearly exhibit quadrant glitch phenomenon as observed from the axial tracking errors and the contour tracking errors of the circular tests. Sliding mode controller reduces quadrant glitches significantly. At the quadrant positions, the radial tracking errors of the three controllers are comparable and are within a range of +/- 10 micrometer.

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