A Comparison of Experimental, Operational, and Combined Experimental-Operational Parameter Estimation Techniques

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Abstract

A modal analysis aims at the identification of the modal parameters of a test structure from the measured vibratory behaviour. Traditionally, both the input forces and the resulting responses are measured. However, in many applications it is not possible to measure (all) the input forces. During the last decade, two new classes of modal parameter estimation techniques have been developed to overcome this problem: the operational techniques and the combined experimental-operational techniques. Operational modal analysis techniques can identify the modal parameters from the responses of the structure; they do not require the input forces. The combined experimental-operational techniques, only require a part of the input forces to estimate the modal parameters.

The work presented in this paper is part of the evaluation process of these new modal parameters estimation techniques; it compares the modal parameters provided by the operational and combined experimental-operational modal analysis techniques with the modal parameters obtained with the experimental modal analysis technique.

Nomenclature

$\diamond^*$	Complex conjugate	j	Imaginary unit
$\diamond^T$	Transpose	L	Modal participation factor
$\diamond^\dagger$	Pseudo-inverse	$\mathcal{P}$	Projection operator
$\{\diamond\}$	Column vector	Q	Modal scaling factor
$\langle\diamond\rangle$	Row vector	λ	Eigenvalue
$[\diamond]$	Matrix	ψ	Mode shape
$h(t)$	Impulse response function	γ	Discrete eigenvalue
H	Frequency response function (FRF)	Ψ	Discrete eigenvector

1 Introduction

Modal analysis techniques allow to measure the modal parameters, i.e. resonant frequencies, damping ratios, mode shape vectors and modal participation factors, of a mechanical structure. With the experimental modal analysis (EMA) approach [1, 2], the structure under investigation is placed in a test set-up where a number of controlled input forces are applied and measured. The response of the structure is measured in a grid of test locations. Although experimental modal analysis is a very versatile technique, it cannot be applied in two important cases. First of all, it cannot be used to analyse a system in operational conditions, e.g. a

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flying aircraft. Furthermore, it is not straightforward to apply an experimental modal analysis on massive structures like highway bridges or oil rigs. Here, the main problem is the power of the excitation device, which is usually insufficient to excite the structure with the required magnitude.

The operational modal analysis (OMA) techniques [3, 4] were developed to overcome the two main shortcomings of the experimental modal analysis approach. The operational techniques only require the response signals of the investigated structure, they do not require the signals of the input forces. Because of this, it is possible to use ambient forces as excitation forces. Typical examples of ambient excitation forces are wind or traffic in the case of a bridge, and atmospheric turbulences in the case of a flying aircraft. Of course, the operational modal analysis approach has a number of limitations. The input forces need to have a uniform spectrum. In case the input spectrum is not flat, the predominant excitation frequencies can appear as system poles in the parameter estimation step, although they are not. Furthermore, the operational modal analysis techniques can identify resonant frequencies, damping ratios and mode shape vectors; the modal participation factors, however, can only be identified if the structure is measured in at least two different test configurations [5].

Currently, a third class of modal analysis techniques is being developed and evaluated [6, 7], namely the combined experimental-operational techniques (OMAX). The goal of these techniques is to combine the advantages of the two previous approaches, by considering two classes of input forces: ambient forces that cannot be measured and artificially applied forces that are measured. In a simplified way, one could state that the ambient excitation forces are used to excite test structure with a sufficiently high amplitude, while the artificially applied forces are used to identify the modal participation factors and, in the case the spectra of the ambient forces are not uniform, to distinguish the actual modes from the spurious modes that are generated by the frequency contents of the ambient input forces.

The work that is described here is part of the evaluation processes of the combined experimental-operational modal analysis techniques. In this paper, the modal parameters obtained with the three modal analysis approaches are critically compared and discussed.

2 Parameter Estimation Methods

In this text, the following three parameters estimation methods are used to identify the modal parameters:

- Poly-reference least-squares complex exponential method (EMA)
- Data driven stochastic subspace identification method (OMA)
- Combined least-squares frequency method (OMAX)

The goal of this section is to provide a summary of the theoretical background of these methods. For each method, references with detailed information are provided.

2.1 Poly-reference Least-Squares Complex Exponential (PLSCE)

In the case of an experimental modal analysis, both the input and response signals are measured. From the measured time signals, the averages of the frequency response functions (FRF) between every input and output location can be computed. There is whole range of methods to extract modal parameters from measured FRFs, however, in this work only the most commonly used method is considered, i.e. the poly-reference least-squares complex exponential method [8]. This method is based on the re-transformation of the FRFs to the time domain which provides an averaged version of the impulse response functions. These impulse response functions can be grouped into an impulse response matrix that is related to the modal parameters as

$$\Big[h(t)\Big] = \sum_{r=1}^{N} \Big(Q_r \{\psi\}_r \{\psi\}_r^{\mathrm{T}} e^{\lambda_r t} + Q_r^* \{\psi\}_r^* \{\psi\}_r^{*\mathrm{T}} e^{\lambda_r^* t} \Big) \tag{1}$$

In the case of sampled data, the o^{th} row of the impulse response matrix, i.e. the row corresponding to the o^{th} output, equals

$$\Big\langle h(n\Delta t)\Big\rangle_o = \sum_{r=1}^{N} \Big(\{\psi\}_{or}\, z_r^n \,\langle L\rangle_r + \{\psi\}_{or}^*\, z_r^{n*}\,\langle L\rangle_r \Big) \tag{2}$$

where n is the sample index, $L_r = Q_r\,\psi_r^{\mathrm{T}}$ are the modal participation factors and $z_r = e^{\lambda_r \Delta t}$ are the complex exponential factors. Since the products of the complex exponentials with the modal participation factors $\big(z_r^{(*)}\;L_r^{(*)}\big)$ are independent of the response station i, they are a solution of the following finite difference equation:

$$z_r^n\,\langle L\rangle_r\,[I] + z_r^{n-1}\,\langle L\rangle_r\,[W_1] + \cdots + z_r^{n-p}\,\langle L\rangle_r\,[W_p] = \langle 0\rangle \tag{3}$$

where p is the equation order which has to be

$$p \geqslant \frac{2n_m}{n_o} \tag{4}$$

in order to find $2n_m$ characteristic solutions, in case there are n_o response stations. Since the impulse responses for the response station o are a linear combination of the characteristic solutions of equation (3), they are also a solution of that equation. Therefore,

$$\langle h(n\Delta t)\rangle_o\,[I] + \big\langle h\big((n-1)\big)\Delta t\big\rangle_o\,[W_1] + \cdots + \big\langle h\big((n-p)\big)\Delta t\big\rangle_o\,[W_p] = \langle 0\rangle. \tag{5}$$

Taking this equation for all response stations simultaneously enables a global least-squares estimate of the matrix coefficients W_1, W_2, …, W_p. Once these matrix coefficients are known, equation (5) can be reformulated into a generalised eigenvalue problem, resulting into $p\,n_o$ eigenvalues z_r, yielding estimates for the system poles λ_r, and the corresponding modal participation factors L_r.

The mode shape vectors can be estimated by considering the following expression for the frequency response functions:

$$H_{oi}(j\omega) = \sum_{r=1}^{N_m} \left(\frac{\psi_{or} L_{ri}}{(j\omega - \lambda_r)} + \frac{\psi_{or}^* L_{ri}^*}{(j\omega - \lambda_r^*)} \right) + UR_{oi} - \frac{LR_{oi}}{\omega^2} \tag{6}$$

where UR_{oi} and LR_{oi} are the upper and lower residual terms which approximate the effects of the modes above and below the frequency band of interest. The left hand side of equation (6) is the measured frequency response function, the right hand side is the modal model where ψ_{or}, UR_{oi}, and LR_{oi} are the remaining unknown parameters. Since the modal model is linear in the unknown parameters, the modes shapes and the residual term can be found by minimising the differences between the modal model and the measured frequency response functions in a least-squares sense. A detailed description and discussion of the poly-reference least-squares complex exponential method can be found in [1].

2.2 Data Driven Stochastic Subspace Identification (SSIData)

In the case of operational modal analysis only the responses are measured. The inputs, which are not measured and thus unknown, are considered to be white noise sequences. In the last decade, several routines have been introduced to extract resonant frequencies, damping ratios and mode shape vectors from response signals. One of the most powerful classes operational modal analysis techniques are the stochastic subspace methods [9]. These methods identify the modal parameters in an indirect way: they identify a state-space model and derive the modal parameters from the identified system matrices. There are two main classes of stochastic subspace methods that are being used to estimate modal parameters: covariance driven methods and data driven methods. The main advantage of data-driven approach is that it does not require any preprocessing of the measured data [3]; the routines identify the models directly from the time signals.

Data driven stochastic subspace identification starts by identifying the system matrices A and C of a stochastic state-space model:

$$\begin{aligned}\{x_{i+1}\} &= [A]\{x_i\} + \{w_i\} \\ \{y_i\} &= [C]\{x_i\} + \{v_i\}\end{aligned} \tag{7}$$

where i is the time instant, y_i the output variable, x_i the state variable, w_i represents the process noise and v_i the measurement noise. In order to cancel out uncorrelated noise, the row space of the future outputs is projected onto the row space of the past outputs. The main theorem of stochastic subspace identification [10] states that the projection $\mathcal{P}_i$ can be factorised as the product of the extended observability matrix O_i and the Kalman filter state sequence $\hat{X}_i$.

$$[\mathcal{P}_i] = [O_i][\hat{X}_i] \tag{8}$$

Using the singular value decomposition of the projection, $\mathcal{P}_i = U_d\, S_d\, V_d$, the state sequence $\hat{X}_i$ can be obtained from the measurement data as

$$[\hat{X}_i] = [O_i^{\dagger}][\mathcal{P}_i], \text{ where } [O_i] = [U_d][S_d^{1/2}]. \tag{9}$$

The system matrices can be recovered from the state sequences $\hat{X}_i$, $\hat{X}_{i+1}$ using the following overdetermined set of linear equations, which is obtained by grouping the state-space models for the time instants i to $i + n_s + 1$, where n_s are the number of samples that are used in the rows of Y_i.

$$\begin{pmatrix}[\hat{X}_{i+1}] \\ [Y_i]\end{pmatrix} = \begin{pmatrix}[A] \\ [C]\end{pmatrix} [\hat{X}_i] + \begin{pmatrix}[W_i] \\ [V_i]\end{pmatrix} \tag{10}$$

Since the state and the output sequences are known and the residuals are uncorrelated with $\hat{X}_i$, the set of equations (10) can be solved, in a least-squares way, providing the system matrices

$$\begin{pmatrix}[A] \\ [C]\end{pmatrix} = \begin{pmatrix}[\hat{X}_{i+1}] \\ [Y_i]\end{pmatrix} [\hat{X}_i]^{\dagger}. \tag{11}$$

The eigenvalue decomposition of the system matrix A provides the discrete eigenvalues and vectors

$$[A] = [\Psi][\Gamma][\Psi]^{-1}. \tag{12}$$

Finally, the modal parameters are obtained from the discrete eigenvalues and vectors as:

$$\lambda_i = \frac{\ln \gamma_i}{\Delta t} \tag{13}$$

$$\{\psi\} = [C]\{\Psi\} \tag{14}$$

As stated in the introduction, the operational modal analysis techniques do not provide an estimation of the modal participation factors. A detailed description and discussion on the data driven stochastic subspace identification method can be found in [3].

2.3 Combined Least-Squares Frequency Method (CLSF)

In the case of combined experimental-operational modal analysis their are two classes of excitation forces: artificially applied forces and ambient forces. The artificially applied forces are measured and thus known; the ambient excitation forces are not measured but are assumed to have a uniform spectrum. The combined experimental-operational parameter estimation techniques thus have to be able to handle both deterministic and stochastic input forces. As already stated in the introduction, these methods are still in the development stage. To the knowledge of the authors, there are currently only two approaches available: a least-squares frequency domain technique [6] and a subspace-based technique [7]. In this work, the least-squares frequency domain approach is considered, simply because it was the only technique for which their was an implementation available at the time.

In the presented work, the combined experimental-operational approach is only used with one measured and one unknown input. For simplicity, the theory will only be presented for this case. A general and more detailed description of the parameter estimation technique can be found in [6]. For the considered case of one measured and one unknown input force, the relation between the response in location o and the input forces can be expressed in the discrete frequency domain as:

$$Y_o(k) = H_o\, F(k) + G_o\, E(k) \tag{15}$$

where k represents the index of the considered frequency line, F is the spectrum of the artificially applied force, E is the spectrum of the ambient force, H_o and G_o are the frequency response functions for the response location o of the artificially applied and ambient force, respectively. Note that since H and G represent the same system, they must have the same physical modes. Using a common-denominator model, expression (15) can be rewritten as:

$$Y_o(k) = \frac{A_o(z_k, a_o)}{B(z_k, b)} F(k) + \frac{C_o(z_k, c_o)}{B(z_k, b)} E(k) + \frac{T_o(z_k, t_o)}{B(z_k, b)} \tag{16}$$

where $A_o(z_k, a_o)$, $B_o(z_k, b)$, $C_o(z_k, c_o)$, and $T_o(z_k, a_o)$ are polynomials in $z_k = e^{j2\pi k/n}$ with coefficients x_1, x_2, ..., x_{n_x} where x = a_o, b, c_o and t_o. In a fist step, the coefficients of the polynomials are identified by optimising a parametric model of the input-output measurements in such a way that the spectrum of the unknown force $E(k)$ is a white noise sequence. This can be achieved by minimisation of the following cost-function:

$$minimise \sum_{o=1}^{n_o} \sum_{k=1}^{n_k} \left| \frac{B(z_k, b)Y_o(k) - A_o(z_k, a_o)F(k) - T_o(z_k, t_o)}{C_o(z_k, c_o)} \right|^2 \tag{17}$$

In a second step, the modal parameters are extracted from the identified polynomials. The poles of the structure, which provides the resonant frequencies and damping ratios, can be found by computing the roots of $B(z, b)$. The mode shape vectors can be extracted from the deterministic part of the input-output relation as

$$\{\psi\}_r = \lim_{z \to z_r} \frac{\{A(z, a)\}}{B(z, b)} (z - z_r) \tag{18}$$

where

$$\{A(z, a)\} = \langle A_1(z, a_1) A_2(z, a_2) \cdots A_{n_o}(z, a_{n_o}) \rangle^{\mathrm{T}}. \tag{19}$$

The modal scaling factor can be obtained from the unit modal a_r scaling model [1] which provides the relation $A_{ii_r} = |\psi_{i_r}|^2$ with,

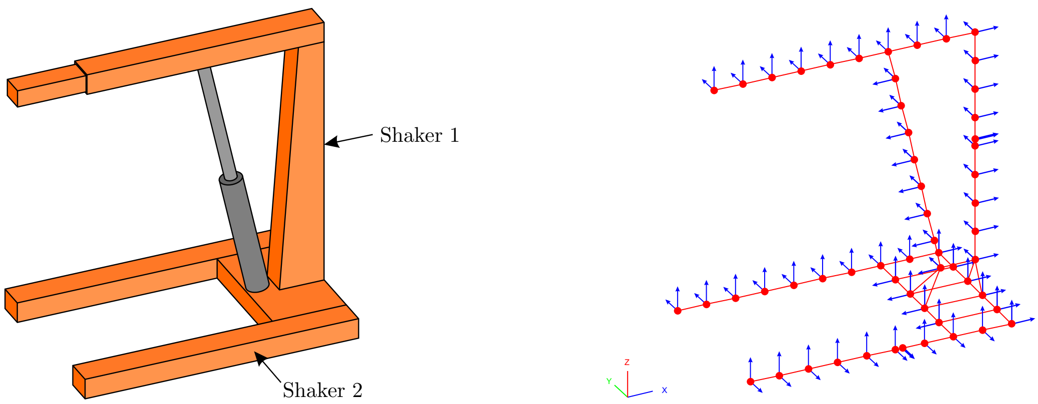
$$A_{ii_r} = \lim_{z \to z_r} \frac{|C_i(z, c_i)|^2}{|B(z, b)|^2}(z - z_r). \tag{20}$$

3 Experimental Comparison

The main goal of this work was to compare the experimental, operational and combined experimental-operational modal testing approaches in an experimental way. A small hydraulic crane was chosen as test structure. To minimise the influence of measurement noise on the comparison, the crane was tested only once and the same data set was processed with the three considered parameter estimation methods.

3.1 The Test Set-Up

Figure 1 provides an overview of the test set-up and the measurement grid. The crane was excited during 4 seconds with 2 electrodynamic shakers that were driven with uncorrelated burst random signals. In the excitation points, the applied force and resulting acceleration were measured with two impedance heads (PCB-288D01). The response of the structure was measured in 54 response points using 8 accelerometers (PCB-A333). In every test point, the response of the structure was measured in two directions, i.e. the two directions perpendicular to the longitudinal axis of the considered component. The response signals were measured during 5 seconds with a sampling frequency of 3200 Hz. No time windows were used and for every measurement 10 realisations were recorded.



(a) The test structure. (b) The test points and DOFs.

Figure 1: The set-up for the modal testing.

3.2 The Parameter Identification

3.2.1 Experimental Modal Analysis (EMA)

The experimental modal analysis was performed with the LMS Test.Lab implementation of the poly-reference least-squares complex exponential method. The FRFs were estimated from the 10 recorded realisations using the H_v estimator. The modal parameters were estimated in the frequency band from 0 Hz to 400 Hz. In this band 28 modes were identified and successfully validated. The modal parameters obtained with the PLSCE method will serve as reference values.

3.2.2 Operational Modal Analysis (OMA)

The structural responses induced by the ambient excitation forces of an operational modal analysis do usually not fade out. Therefore, it was decided to remove the last second of the recorded time signals, i.e. the decaying part resulting from the burst random excitation. The parameter estimation was performed with the data driven stochastic subspace implementation of the department of civil engineering of the Katholieke Universiteit Leuven. With this software, a trade-off had to be made between the number of processed response stations and the maximum model order in the stabilisation diagram: a higher number of response stations resulted in a lower maximal model order. Because of this, the test data were processed in two different ways:

OMA-1: All the measured response signals were grouped and processed as a single measurement batch. The stable poles had to be estimated from a stabilisation diagram that had a limited model order.

OMA-2: The measured response signals were separated into four batches that were processes separately. The shaker signals were used in all four batches and served as reference signals to combine the four partial mode shape vectors into to the full mode shape vector.

The plots of figure 2 provide an overview of the comparison between the modal parameters of the experimental and operational modal analysis. In general, the following observations were made:

- The resonant frequencies and damping ratios of OMA-2 matched better with the EMA results than the resonant frequencies and damping ratios of OMA-1. Processing the data in separate batches thus provided more accurate estimates of the resonant frequencies and damping ratios.
- The mode shapes of OMA-1 matched better with the EMA results than the mode shapes of OMA-2. Processing the data in separate batches appeared to have a negative influence on the quality of the mode shapes.
- There was a clearly better match between the absolute values of the EMA and OMA mode shapes than between the actual mode shapes.

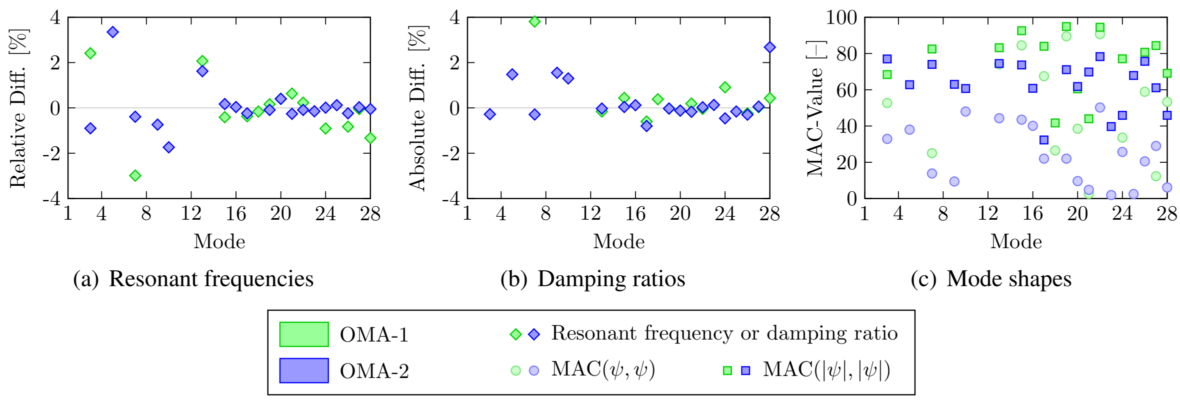


Figure 2: The correlation between the EMA and OMA results.

3.2.3 Combined Experimental-Operational Modal Analysis (OMAX)

The combined experimental-operational modal analysis was performed with the combined least-squares frequency implementation of the department of mechanical engineering of the Vrije Universiteit Brussel. For the same reason as for the operational modal analysis, it was decided to remove the last second of the time signals. The data was processed two times:

OMAX-1: The input force of shaker 1 was considered to be known, while the input force of shaker 2 was considered to be unknown.

OMAX-2: The input force of shaker 2 was considered to be known, while the input force of shaker 1 was considered to be unknown.

The plots of figure 3 provide an overview of the comparison between the modal parameters of the experimental and combined experimental-operational modal analysis. In general, the following observation were made:

- There was no clear difference in quality between the modal parameters of the two OMAX estimations. Both sets of OMAX modal parameters provided a comparable level of correlation with the EMA results.
- There was a better match between the absolute values of the EMA and OMAX mode shapes than between the actual mode shapes.
- The mode participation of the EMA modes revealed that the contribution of the second shaker to the excitation of modes 1, 2 and 4 was more important than the contribution of the first shaker. However, these three modes were only found by OMAX-1, not by OMAX-2.

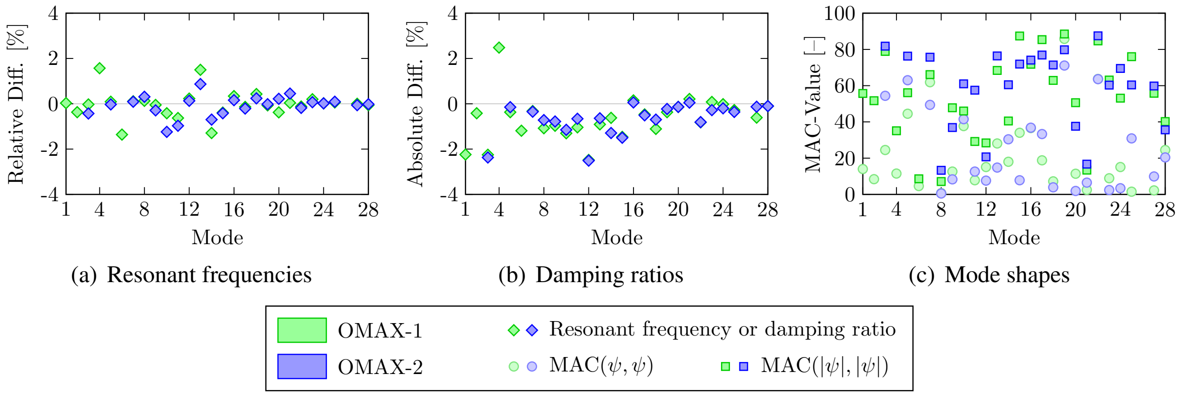


Figure 3: The correlation between the EMA and OMAX results.

3.3 Discussion

Table 1 provides an overview of the correlation between the EMA, OMA and OMAX results. Based on this information, the following conclusions can be drawn:

- Both OMA and OMAX did not result in a systematic over- or underestimation of the EMA resonant frequencies.
- The OMAX resonant frequencies correlated better with the EMA resonant frequencies than the OMA resonant frequencies.
- The OMA damping ratios were an overestimation of the EMA damping rations, while the OMAX damping ratios were an underestimation of the EMA damping ratios.
- The OMA damping ratios correlated slightly better with the EMA damping ratios than the OMAX damping ratios.
- The correlation with the EMA modes shapes was very poor for both the OMA and OMAX modes shapes.

Table 1: Summary of the correlation between the modal parameters of EMA, OMA and OMAX.

	OMA-1	OMA-2	OMAX-1	OMAX-2	
Identified modes	22/28	27/28	27/28	23/28	
$\sum(\Delta f/f)/n$	-0.08	0.04	-0.02	-0.08	[%]
$\sum(\|\Delta f\|/f)/n$	0.93	0.56	0.37	0.31	[%]
$\sum\Delta\delta/n$	0.38	0.25	-0.65	-0.68	[%]
$\sum\|\Delta\delta\|/n$	0.57	0.53	0.87	0.67	[%]
$\sum \text{MAC}(\psi)/n$	50.8	23.8	22.3	25.0	[–]
$\sum \text{MAC}(\|\psi\|)/n$	75.5	61.3	53.8	59.2	[–]

- The correlation of the mode shapes improved if the absolute values of the modal displacements were used. This indicates that, both OMA and OMAX, provided a better estimation of the amplitude than of the phase the modal displacements.

A more detailed discussion on comparison of the identified modal parameters can be found in reference [11].

4 Conclusions

This paper presented a comparison between the following three modal testing approaches: experimental modal analysis, operational modal analysis and combined experimental-operational modal analysis. These three techniques were compared in an experimental way by using a set of test data that was obtained on a small hydraulic cane. The same data set was used to identify the modal parameters with a representative technique of the three modal testing approaches. After comparing the identified modal parameters, a number of conclusions were drawn.

Note that the conclusions that are presented in this paper should be used with caution. More research is needed to determine whether the conclusions that are presented here are valid in general, or whether they are only valid for the considered test structure or for the selected parameter estimation routines.

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