The Fuzzy Parameterization Method for Model Updating Application to Welded Joints

O. Ait-Salem Duque [a], **M. De Munck** [b], **A. Stenti** [b], **D. Moens** [b], **A. R. Senin** [c]

[a] INSIA (University Institute for Automobile Research).
Polytechnic University of Madrid. Ctra Valencia km7,
Campus Sur UPM 28031. Madrid. Spain.
e-mail: **omar.aitsalem@upm.es**
[b] K.U.Leuven, Department of Mechanical Engineering,
Celestijnenlaan 300 B, B-3001, Heverlee, Belgium
[c] Department of Mechanical Engineering. Nebrija University. Pirineos, 55. 28040 Madrid. Spain.

Abstract

The use of partly rigid beams for modelling welded joints has been used and accepted by many authors but some major problems that encounter this type of parametrization, like the determination of the initial values of the offsets, still haven't been solved. A new method for subselecting the parameters and its corresponding nominal values is presented in this paper. The novel aspect of this approach is that the procedure makes use of the Fuzzy Finite Element method as a reliable tool for finding the best possible initial conditions of the parameters before updating an FE model, aiming to cope with two of the main problems of model updating: The undetermined problems occurring when the number of parameters outweigh the number of available test data in one hand, and, avoiding the convergence of the solution to a local minimum on the other hand. The method is applied to welded joints which are always a source of uncertainty and that are commonly used in big automotive structures like the body frames of buses.
Keywords: model updating, interval finite elements method, fuzzy finite element method, welded joints, offsets...

1 Introduction

An FE model of an automotive joint is generally defined as an intersection region of beam elements, this is mainly due to the fact the use of shell elements in an FE model is very expensive computationally speaking. In the case of welded joints, the use of coupled rotational springs is very commonly used, but in the case of body frames of buses the stiffness of the joints is too high to consider the use of springs, so the joints are considered to be rigid, the welding process introduces an extra stiffness that some how must be taken into account. J. Wang and P. Sas [1] presented some models of mechanical joints which are suitable for FE dynamic analysis, as well as their parameterisation based on experimental modal results.

The use of Finite Element model updating, is nowadays commonly accepted as a reliable tool to improve the correlation between the experimental dynamic response of a structure and the corresponding predictions of the analytical model. A considerable number of researchers have focused their work on generating and testing different updating methods over the past *25* years (see i.e. [2] and [3]). There are several possibilities to classify the different algorithms based on whether they work in the frequency or modal domains and whether they adjust the mass and stiffness matrices directly ”direct methods” or make parametric changes to the model ”indirect or parametric methods”. It has been proved that direct methods are not appropriate for model updating because they are not able to provide physically meaningful results (e.g see [4]).

J.E. Mottershead et al. [5] applied the sensitivity method to update finite element models of welded joints. They found out that careful parameterisation is critical in updating this particular type of joint. The use of nodal offset was shown to result in an updated joint with a physical interpretation. Keeping the physical meaning of the model, is one of the main objectives in model updating because a random change in the parameters could lead to a model that matches the experimental results but which is far away from the real structure.

The fuzzy finite element method for the calculation of frequency response functions (FRFs), developed by D. Moens [6], combines the concept of fuzzy sets with the modal superposition principle. This method calculates large scale sensitivities of the response of the structure to the uncertain input parameters.

2 FE Updating: The Eigensensitivity Approach

In this paper the method for Updating the FE model that will be followed is a sensitivity matrix based approach, which minimises the difference between modal quantities (usually natural frequencies and less often mode shapes) of the measured data and model predictions. This problem may be expressed as the minimisation of a cost function J whose expression was formulated by M. Friswell [7] as:

$$J = \epsilon^T.W_{\epsilon\epsilon} + \alpha.\delta\theta W_{\theta\theta}.\delta\theta \tag{1}$$

Where $\epsilon = \delta z - S^j.\delta\theta$ is the modal residual vector. The weighting matrices $W_{\epsilon\epsilon}$ and $W_{\theta\theta}$ represent the analysts confidence in the initial model parameter values and the accuracy of measured data respectively, the parameter α controls the regularisation due to the initial parameter values and $\delta\theta$ represents the infinitesimal variation of the parameters chosen during the parameterization process. In the expression of the modal residual vector ϵ, $\delta z = z_m - z_c$ is the difference between the measured z_m and computed z_c modal parameter vectors and and S^j represents the sensitivity matrix consisting of the first derivatives of the modal quantities with respect to the model parameters. The cost function of equation (1) is a non-linear function of the parameters and the minimisation is solved using a truncated linear Taylor series and iteration. At each iteration of the model updating procedure, the parameters are changed according to equation (2). In (3), n represents the number of identified natural frequencies, d the number of parameters chosen during the parameterization process and j denotes the j^{th} iteration.

$$\delta\theta = \left[S^T.W_{\epsilon\epsilon}.S + \alpha.W_{\epsilon\epsilon}\right].S^T W_{\epsilon\epsilon}\delta z \tag{2}$$

$$\delta z \in R^n, \delta\theta \in R^d, S^j \in R^{n\times d} \tag{3}$$

$$S_r^j = \frac{\delta\lambda_j}{\delta\theta_r} = \phi_j^T.\left[\frac{\delta K}{\delta\theta_r} - \lambda_j.\frac{\delta M}{\delta\theta_r}\right].\phi_j \tag{4}$$

In equation (4) is given the expression of the sensitivity matrix. Where $\lambda_i = w_j^2$ is the j^{th} eigenvalue and ϕ_j is the j^{th} eigenvector. In [7] Friswell and Mottershead give more detail on the algorithms available for model updating.

In this paper mode shapes will not be used for the updating studies, except for pairing individual modes.

3 Presentation of the Problem

The Fuzzy Finite Element method (FFE) for the calculation of frequency response functions (FRFs), developed by D. Moens [6], combines the concept of fuzzy sets with the modal superposition principle. This method calculates large scale sensitivities of the response of the structure to the uncertain input parameters.

The goal of this work is to implement the FFE method as part of the model updating procedure of welded joints, where the parameters chosen for the updating are the offsets of the partly rigid beams used to model the welded joints, as described in reference [8]. The implementation of the new approach will be at three different levels:

- Large scale Sensitivity analysis.
- Subset selection of parameters.
- Definition of the initial values of the offsets, to be used in the updating process.

The conventional model updating procedure, shown in figure (1), generally presents two problematic aspects. The first aspect is related to the risk of falling into unwanted local minimums of the cost function even in case of convergency. This give rise to the question: what is the best possible combination of initial values for the chosen parameters that could make the updated model closer to the real structure?

The second aspect is related to the fact that the sensitivity analysis is only correct in the surroundings of the initial *correct* parameter values. A typical case is shown in figures (2), where θ and λ represents respectively the parameter and the eigenvalue.

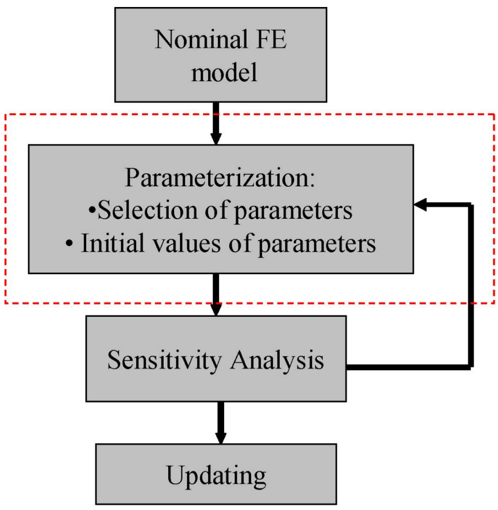

Figure 1: The Model Updating Procedure

By focusing on point *2* in figure (2), a classical conclusion following the conventional model updating procedure would be that since the sensitivity of the eigenvalue in the sorroundings of the parameter is very low, this parameter can be neglected in the updating procedure. We see however that by simply chosing a different initial value, i.e. *1* or *3*, our drawn conlcusions would have been different.

If we go further in this reasoning, we can think of another case (case2) shown in figures (3) and (4)by comparing these two figures we can draw some more interesting conclusions. If we had to choose between the two parameters with initial values in point *2*, we would definitely go for *Case 2.2*, because here the slope in point *2* is bigger than in *Case 2.1*, where it is almost zero. But, if we take a closer look to the global change

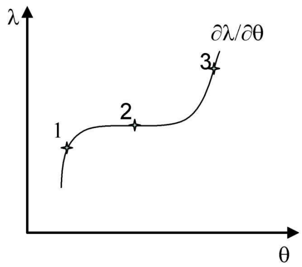

Figure 2: Case 1

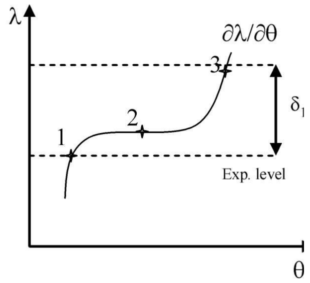

Figure 3: Case 2.1

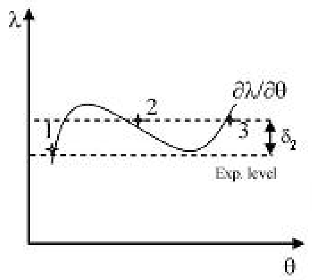

Figure 4: Case 2.2

of the eigenvalue within the whole range of interest (along the θ axis), we see that, the change in *Case 2.1* is bigger than in *Case 2.2* ($\delta_1 >> \delta_2$). So our first choice, based on the change of slope in point *2* (as in the conventional sensitivity approach) would have been wrong, while a selection made based on a detailed look at a wider interval (Interval method) could lead us to a more realistic conclusion.

The proposed procedure in this paper is based on the use of the FFE method to obtain the best possible set of parameters all along with their respective initial values, before starting the updating process.

4 Case-Study: Part of a Bus Body Frame

4.1 Experimental Structure

The structure that is going to be studied in this work is part of the lateral of the body frame of a bus. The bus is a urban type. The structure is made up of different hollowed section steel beams. The tested structure is shown in figure (5) and a description of the main characteristics of each joint beam is shown in figure (6).

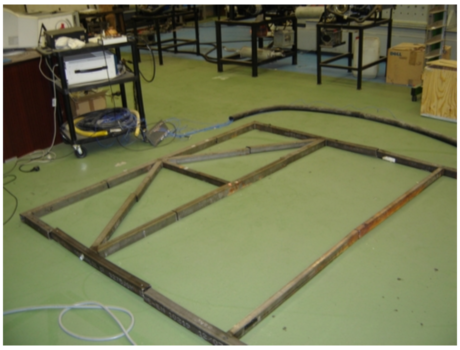

Figure 5: The Experimental Structure

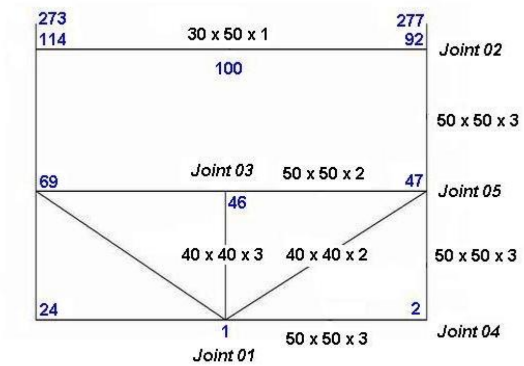

Figure 6: Geometry of the Experimental Structure: $mm \times$mm $\times mm$

The structure is subjected to several modal tests under free-free conditions in order to extract its dynamic characteristics. The results of the experimental modal analysis are used as a reference in the modified model updatig procedure. The structure is excited at different locations, but for the procedure only the point FRF at node *2,* in the *z* direction (out of plane) will be used. Figure (7) shows the measured point FRF and its synthetised counterpart, the curve fitting can be considered good enough to take the experimental results as a starting point. The modal parameters extracted from the experimental modal analysis are used later for the superposition with the analytical fuzzy results, some of the modes like modes *1*, *3*, *5* to *8* and *10* are in the *z* direction while modes *2* to *4* and mode *9* are in plane modes for the structure.

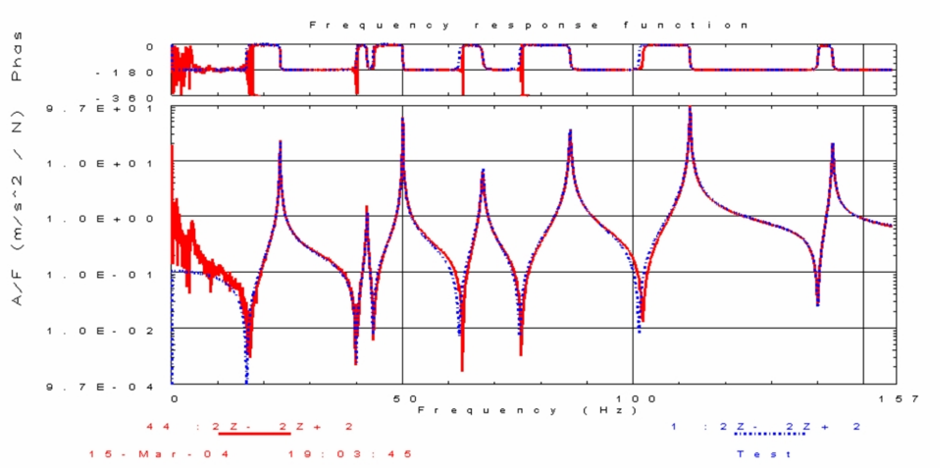

Figure 7: FRF experimental vs. synthesized

4.2 Modelling Considerations: Baseline FE model

All the elements used in the FE model are 3D beam elements with six DOF at each node. The FE model has in total 69 elements and 65 nodes. Five kinds of different joint types were identified (see figure (6)). Each joint is assumed to be symmetric in order to minimise the number of parameters employed in the study, ad is modeled used the concept of partly rigid beams, as introduced by Ahmadian [8]. As it is shown in figure (8), a beam of total length *L* is made partially rigid in proximity of the joints to which it is attached. The length of the rigid part, namely *a*, is the parameter used in the updating procedure. For each joint, depending on whether it is a horizontal, vertical or diagonal beam a different parameter is chosen.

Joint 01 shown in figure (9) has three parameters *ah01*, *av01* and *ad01* and the same applies for the rest of the five joints, making an overall of *19* parameters. Table (2) summarises the results from both the experimental and the analytical study, and quantifies the error at each eigenvalue before the updating process. The error, as in the subsequent tables, is calculated as $(\lambda_{Exp} - \lambda_{Anal}) / \lambda_{Exp}$.

In the updating procedure the focus will be on the modes showing the higher percentage of error (highlighted in table (2)), assuring that the introduced variation in the updating parameters will not result in a deterioration of the initial good performance of the model in the other modes.

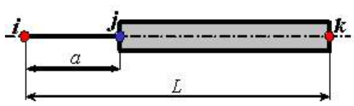

Figure 8: Partly rigid beam

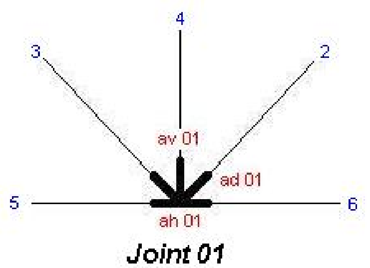

Figure 9: Parameterisation of Joint 01

4.3 Description and Application of the Methodology

The alternative procedure for model updating introduced in this paper is made of three phases. In the first phase, the *Subset Selection* phase, the aim is to reduce the number of parameters used for the updating procedure. It is very important to have less parameters than experimental results to avoid undetermined systems in the updating process. In the *Initial Value* phase, an optmisation is run with the aim to find the best possible starting values for the last *Model Updating* phase.

4.3.1 Subset Selection

The IFE method is used for a preliminar large sensitivity analysis in order to assure, prior to the optmisation phase, that the experimental measurements lays within the possible bounds of variation and the updating process is capable of operating on all the interested peaks of the structure. For this analysis, the 19 identified parameters have a zero intial values and a maximum variaton lenght corresponding to the FE model mesh size, in proximity of the interested joints.

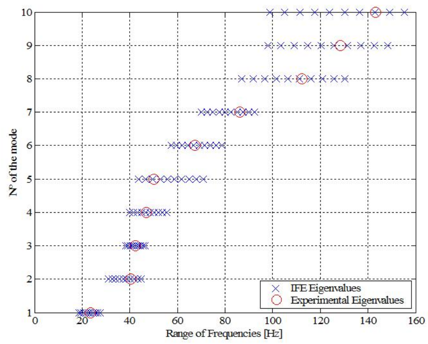

Figure 10: FRF experimental vs. synthesized

Figure (10) shows the results in terms of frequencies. It can be seen that the *10* experimental modes are inside the bounds of variation resulting from the interval analysis. Some of the modes show very little dispersion around the corresponding experimental frequency like modes *1* and *3* while modes like *9* and *10* experience a wide variation for the different combinations of the parameters values.
Once the large sensitivity analysis has been performed, and having ensured that the experimental eigenvalues are within the bounds of the output FRFs generated from the IFE analysis, the subset selection phase starts. The numbers of available experimental results are *10* and we have identified *19* parameters for the updating. The subset selection phase will reduces the parameters from *19* to *6*, by running *19* independent analyses, one for each parameter, to verify the influence of each chosen parameter on each mode of the structure. For each analysis one parameter is centered around zero with a maximum variaton lenght corresponding to the FE model mesh size in proximity of the correspondent joint.
We are implicitly assuming that the maximum influence on each of the modes is assured by the procedure. The joint effect of the parameters will be evaluated in the *Initial Value* phase, only for the selected subset.
In figures (11) and (12) two cases are shown related to mode *8*, which is one of the modes that have more than *5 %* of error with respect to the experimental results. Parameter *ah02* has definitely more influence on mode *8* than parameter *av01*, which in addition does not induce any relative change in none of the ten modes. This reasoning make us to discard parameter *av01*. Parameter *ah02* on the other hand, has a clear influence on mode *8* and it keeps the other modes unaltered; this makes parameter *ah02* a good parameter candidate to be chosen in the updating process. The best candidates however will be the parameters which will have the most influence on the modes with the higher percentage error to the experimental results, leaving unhalterd the initial good performance of the model in respect to the other modes. In table (2) are highlighted the modes on which we will focus in the updating procedure. These are modes: *1, 2, 4, 7* and *8*. At the end of

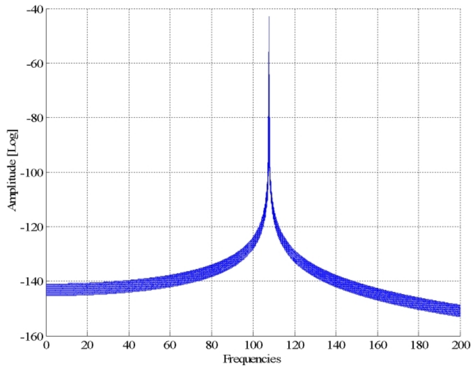

Figure 11: mode *8*, parameter *av01*

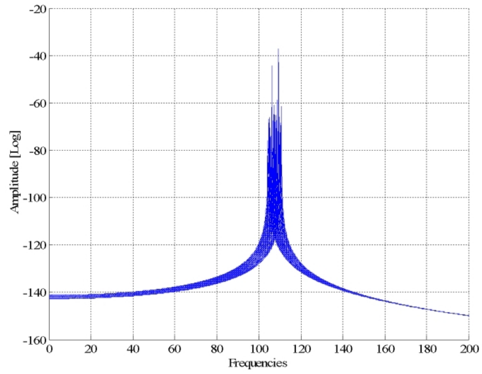

Figure 12: mode *8*, parameter *ah02*

the subset selection phase, *6* out of *19* parameters have been selected as the best candidates for the updating process. The selected subset, together with their initial values is shown in table (1).

4.3.2 Initial value selection

Using the selected subset as a whole, a first optimisation is performed in order to extract the best $\alpha - level$ which keeps the experimental eigenvalues inside the range of variation of the analytical ones. A second optimisation is then performed independently on each parameter of the subset, using the $\alpha - level$ obtained in the previous optimisation as a starting point. In this phase, by using asymmetric triangular membership functions for the interested parameter and an appropriate rectangular membership function for the other parameters, the procedure should lead to best initial value for each parameter, to be used in the updating procedure.

Determination of the optimum $\alpha - level$. In the first optimisation a triangular membership function is used (see figure (13)). Five $\alpha - levels$ are used for the calculation. Starting from the $\alpha_0 - level$ the optimum would be to be able to reduce the $\alpha - levels$ up to the peak value assuring that the eigenvalues of the analitical simulation converges through the experimental values. In this case the mean values of the triangular membership functions would represent directly the best initial values for the updating process and there would be non need in using the asymmetric triangular membership funtion of the second optimisation. The results of the first optimisation process are shown in figure (17) where the envelope FRFs are shown together with their relative membership value. An additional comment can be made on the amplitudes of the various modes. As the experimental values in table (2) have been used as the damping factors, a good match in the amplitude values is clearly visible.

In figure (16) more insight is given to the evolution of the bounds for each peak. The $\alpha_{0.75} - level$ turned to be the limit above which the analytical eigenvalues where crossing the experimental ones. The envelope FRF relative to the $\alpha_{0.75} - level$ is shown in figure (18).

Determination of the initial values. The procedure, summarized in figure (15), emphasize the convenience of using asymmetric triangular membership functions to identify the best possible initial values for the selected subset of parameters. A total of *14* independent analyses, two for each parameters, are performed. In the first analysis the first parameter will show an asymmetric left triangular membership function with the $\alpha_0 - level$ corresponding to the $\alpha_{0.75} - level$ previously calculated; while all the other parameters will show a rectangular membership function with a costant width also corresponding to the previously calculated $\alpha_0 - level$. The second analysis will have the first parameter showing an asymmetric right triangular membership function with the $\alpha_0 - level$ corresponding to the $\alpha_{0.75} - level$ previously calculated; while all the other parameters will show a rectangular membership function with a costant width corresponding to the previously calculated $\alpha_0 - level$; and so on for the remaining parameters. Once the fourteen independent fuzzy analysis are performed the final values are chosen according to the following criteria, highlighted in

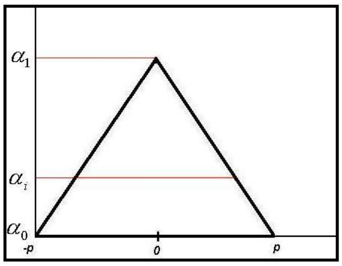

Figure 13: Triangular membership function

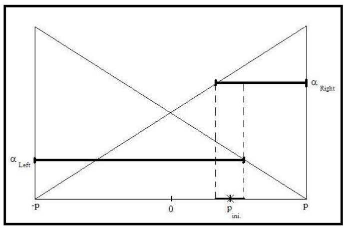

Figure 14: Procedure for the determination of the initial parameter values

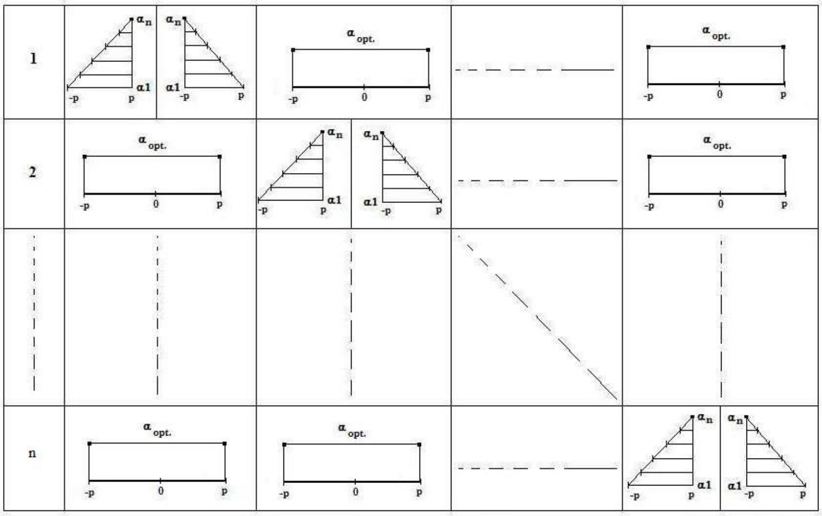

Figure 15: Membership function families for the determination of the initial parameter values

figure (14):

- Selection of the optimum $\alpha - level$ of the left triangle for each parameter.
- Selection of the optimum $\alpha - level$ of the right triangle for each parameter.
- Extraction of the intersection interval.
- Picking the mean value of the previous interval as the initial value.

The result of the process is summarised in table (1), where the initial lengths of the offsets selected as candidate parameters are shown.

4.3.3 Model Updating

The previously chosen subset with the corresponding calculated initial values are used in the final phase, the updating process. The selected *6* parameters together with the *10* experimental modes are used for the updating process. The results are shown in figure (19) and summarized in (2). It can be appreciated an improvement of modes *1*, *2*, *4*, *7* and *8*, that were highlighted previously as the goal of this procedure. Modes *3*,*5*,*6* have not been improved, but they have been kept in good agreement with the experimental results: less than 5% of error. The final values obtained for the *6* parameters, are shown in table (1).

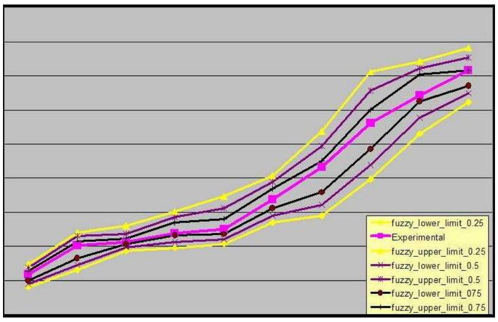

Figure 16: Evolution of the bounds with increasing $\alpha - levels$ in comparison with experimental eigenvaluests

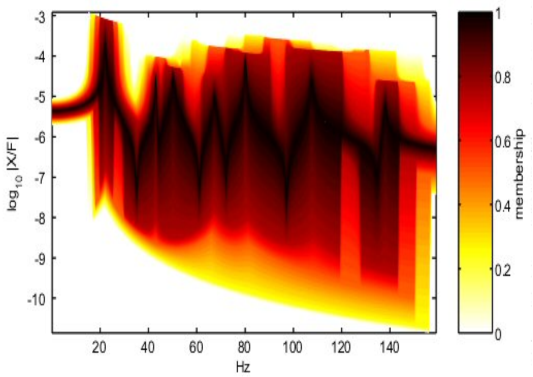

Figure 17: Fuzzy analysis results with 6 parameters

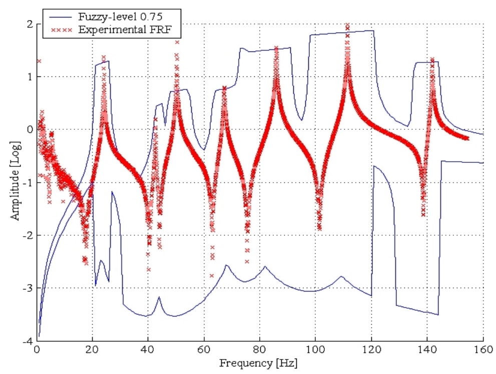

Figure 18: Superposition of experimental and fuzzy FRF's with $\alpha_{0.75} - level$

In the figures (20) and (21) two different situations are highlighted, both related to the typical situation of lack of information on initial values of the updating parameters. In figure (20) it is shown that if we set all parameter values to zero, the updating process is stopped after only four iterations, even if there is a reduction of the error on the eigenvalues, the model has become non physical, because the length of some of the parameters have become larger than the actual mesh size of the elements close to the updated joints. In figure (21), it is shown on the other hand that if we choose random numbers as initial values for the six parameters, the updating process diverges which means that clearly the target objective has been missed, and that a bad choice of the initial values could cause the hole process to diverge.

It is clear that having good intial estimation of the parameters is crucial to success in having a correct updated

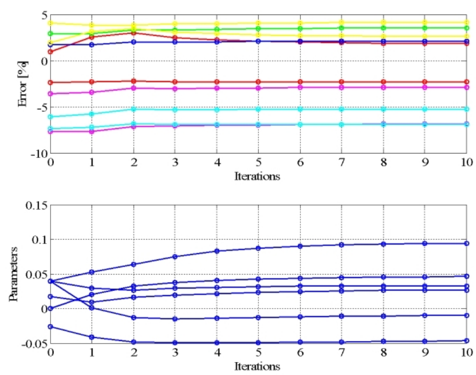

Figure 19: Evolution of the errors and the parameter values in the updating process.

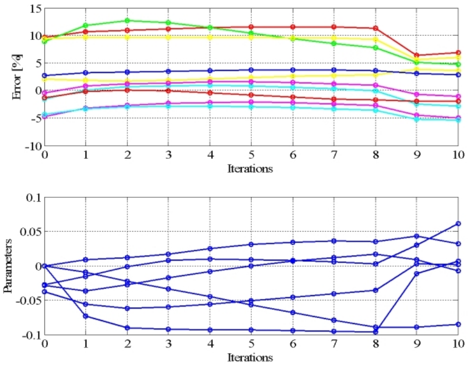

Figure 20: Example of divergency due to random initial values

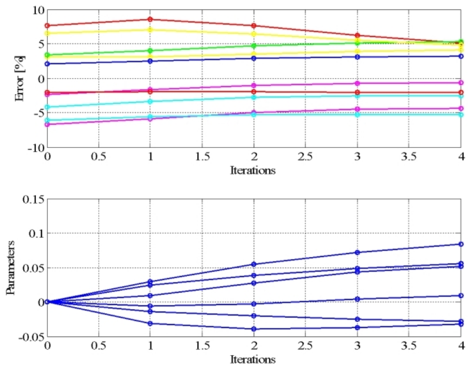

Figure 21: Example of non-physical model due to nominal model with zero as initial values.

Parameter	Name	Inital values	Final values
1	ah02	0	0.0943
2	av02	0.039	0.0464
3	ah04	0.039	-0.0105
4	av04	0.039	-0.0466
5	ah05	- 0.026	0.0325
6	av05	0.039	0.0265

Table 1: Selected Subset, Initial and Final values

finite element model of a real structure.

5 Conclusions

In this paper a new methodology has been described. The objective was to have the best possible initial conditions before an updating process of a finite element model is performed, by using the fuzzy finite element method. And taking advantage of the fact the fuzzy method allows larger variations of the parameters, than the standard sensibility analysis. The improvement of the updating process is at three levels, the first of them at the level of the sensitivity analysis, the second is at the level of the subset selection of the parameters allowing the reduction of their number, so that a reasonable number of experimental eigenvalues can be used. And the third level is the estimation fo the initial parameters, which has been proved to be crucial for the success of the updating process, preventing from falling into non-physical models or parameters which do not converge. The methodology has been applied to a real structure of the lateral of a bus made of welded joints, in order to see its application in a realistic industrial field like the manufacturing of buses.

		Before Updating		After Updating	
Mode	$\lambda_{Exp}[Hz]$	$\lambda_{Anal}[Hz]$	Error [%]	$\lambda_{Anal}[Hz]$	Error [%]
1	23.54	22.23	**5.90**	22.31	**5.23**
2	40.42	37.34	**8.27**	37.65	**6.86**
3	42.36	43.09	1.71	44.11	-4.13
4	47.71	50.30	**5.16**	48.59	**-1.85**
5	50.11	50.93	1.62	51.88	-3.54
6	67.46	67.48	0.03	68.88	-2.10
7	86.40	80.08	**7.89**	80.43	**6.91**
8	112.34	107.54	**4.46**	109.09	**2.89**
9	128.56	134.37	4.33	131.92	-2.61
10	143.26	137.76	3.99	139.95	2.31

Table 2: Differences between nominal and experimental modes

The results of the work are encouraging but the methodology still needs to be improved, to have more accurate initial values, and even, if applied iteratively, as an actual updating tool to that uses both the eigenvalues and the FRFs of an experimental structure, subjected to modal testing. Other parameters like cross section areas of the beams and their thicknesses should also be tested with the methodology to compare their sensitivity to the one of the offsets parameters.

The intention of the authors is to apply the present methodology to other types of structures with different materials and geometries to ensure that it works under different circumstances.

6 Acknowledgement

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