# Reduced modal models and negative concept modifications in dynamic analysis

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## Abstract

In this paper a concept CAE approach to efficiently modify the properties of beam-like sections and joint connections of a vehicle body is presented. Negative concept modifications in the beams and in the joints are analyzed using reduced models. Standard beam elements are used to implement the modifications in the beam-like section, where the joint modifications are considered through Guyan superelements. Examples are presented for beam thickness and joint stiffness modifications, but one can consider any properties: it will be shown that by adding beams to the beam model and by applying Guyan superelements to the joint model one thus obtains a much smaller models for fast modification analysis. The proposed approach is then demonstrated on an industrial vehicle model to quickly and accurately optimise the low-medium frequency behaviour.

## 1 Introduction

In order to achieve a true "Design Right First Time" which leads to shorter time-to-market and reduced costs as compared to conventional "Test Analyze & Fix", one must apply predictive Computer-Aided-Engineering (CAE) methods in all stages of the design process. A major challenge and ongoing revolution in digital product development consists of achieving an "Analysis leads Design" process, in which an upfront engineering analysis phase essentially precedes the detailed (geometrical) design (CAD and CAE) and in which CAE supports concept analysis to define the design requirements in order to meet the functional performance targets.

For this purpose, the authors have developed a concept CAE approach, also known as simplified modelling, which uses approximations that reduce the size and complexity of the large FE model. The concept models are characterized by subdividing the structure into beam-like and joint-like components as shown in Figure 1. The properties of the beam-like structures are obtained directly from the refined FE model, and then represented by equivalent beam elements. Similarly, the stiffness characteristics of the body joints (between the beam-components) are derived from the FE model, and the joints are represented by small-sized static superelements (system matrices).

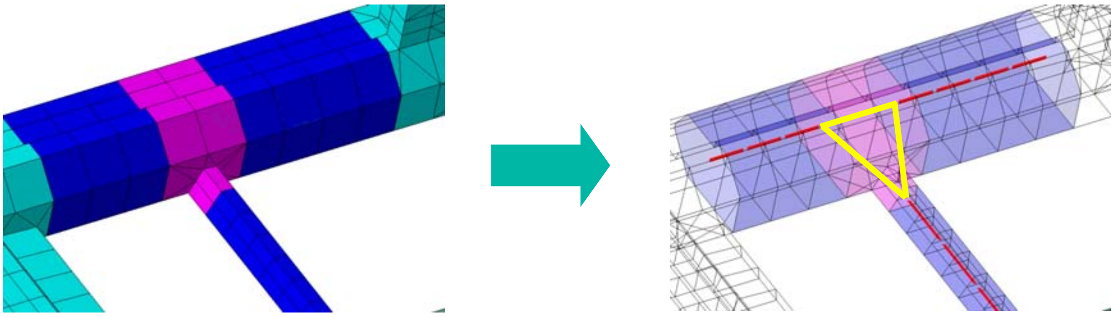

**Figure 1: Example of process for creating finite element concept models: Beam (---) and Joint (Δ) representations.**

An overview of positive beam and joint modification analysis scenarios is presented in Section 2. Sections 3 and 4 present the negative modification approaches for beam-like sections and joint connections. Up till now, the procedures have been implemented for the Nastran [1] FE solver, but these can be developed also for other solvers in a straightforward manner. An industrial application on a racing vehicle chassis, for which modifications and optimisations are performed, is presented in Section 5. The paper is concluded in Section 6.

## 2 Positive modifications

In this section positive beam and joint modifications will be presented. The positive modification analysis has already been performed in [2] but the procedure and results will be briefly explained here. The primary structural members are beam-like structures with a length much greater than the characteristic dimensions of the cross-sectional area. These primary beam members, typically modeled with shell elements, determine the vehicle's fundamental eigenfrequencies. A beam modification analysis tool has been developed to efficiently modify and optimize the fundamental vehicle dynamics. For each primary beam member cross-section, the equivalent beam properties are computed and then taken as equivalent representation of the primary beam member (more details can be found in [2]). When an equivalent beam layout has been obtained, the beams can be re-scaled and added to the original model in a **beam positive modification analysis** framework. As example modifications for the beam-like members, shell thickness modifications are considered in this paper. Three methods exist to apply positive modifications to the beam member. One can directly *modify the original FE model* or one can *add beams to the original FE model* or *to a reduced modal model*. In this last method one has to create a reduced modal model, on the beam center nodes (i.e. the nodes in the geometrical center of the beam cross-section), scale the equivalent beam properties and apply these beams between the beam center nodes. All three methods have been tested in [2] for a shell thickness modification of 50% on the sample beam in Figure 2. It can be seen [2] that the fundamental (first 6) modes are accurately predicted by adding (appropriately scaled) equivalent beams to the reduced modal model at 100% of the nominal properties.

**Figure 2: Cylindrical beam (left) and joint (right) modeled with shell elements.**

Complementary to the beam modification analysis tool, a **joint modification analysis tool** has been developed to modify the stiffness of joints between primary structural members in a vehicle. Guyan reduction [3] is used to compute a static superelement that contains stiffness relations between the end points of the joint (i.e. the beam center nodes). A similar reduction of the mass matrix is not considered in this paper. In the joint model the stiffness modification is applied by adding the scaled stiffness matrix between the end points of the joint to the reduced modal model of the joint. In analogy with the beam positive modification analysis, one can distinguish three methods to perform the positive modification analysis at joint. One can *modify the shell stiffness (Young's modulus) in the original FE model*, or one can *apply static Guyan superelements to the original FE model* or *to the reduced modal model*. In fact, for a stiffness modification range up to 50%, it has been verified in [2] that the Guyan superelements are statically equivalent to the original modifications, and that also the low and medium frequency range (up to 700 Hz) are accurately predicted.

Therefore, by adding beams to the reduced models and by applying Guyan superelements to the joint models, one thus obtains a much smaller model with an easy parameterization without loss of accuracy.

# 3 Negative modifications at beams

In analogy with positive modification, one can consider **negative concept modifications**. As before, shell thickness modifications are considered. For beam members, three methods for negative modifications are distinguished:

A. **Modify the original FE model**: Create a property group for the beam member and change (decrease) its properties (conventional method).

B. **Add beams with negative properties to the reduced modal model (or to the original FE model)**: Scale the equivalent beams to accurately represent the negative modifications in the original model. Add the scaled beams to the reduced modal model (or to the original FE model). It will be explained below that this method is not applicable for beams.

C. **Add beams with positive properties to the modified reduced modal model:** First modify the original FE model by decreasing the beam properties as in Method A. For this modified FE model, create a reduced modal model from the normal modes solution ("modified reduced modal model"). Scale the equivalent beams (obtained from the nominal, non-modified FE model) and add the scaled beams to the modified reduced modal model.

Method A. is the conventional method, but it is quite tedious and time-consuming to manually modify the FE model properties. Method B. is interesting but not applicable, because it is not allowed to define negative values in beam properties (e.g. Nastran card PBEAM). Method C. is an efficient method because the beam modification is easily applied on a small-sized reduced modal model, for which the structural analysis results are quickly obtained. By first decreasing the beam properties, and then applying beam modifications on the modified reduced modal model, one can explore the negative modification range (w.r.t. the nominal FE model) through positive additions (of equivalent beams to the modified reduced modal model).

Accurate results are obtained with Method C. Figure 3 shows the comparison of Method A. with Method C.. The comparison is made between the original FE model at 100% (i.e. nominal properties) and the reduced modal model at 70% of nominal properties, to which beams with positive properties at 30% are added. It can be seen that the fundamental (first 6) modes are accurately predicted by adding equivalent beams to the reduced modal model. Here, no deformations in beam-member cross-sectional areas occur, so that beam modifications are applicable in this range.

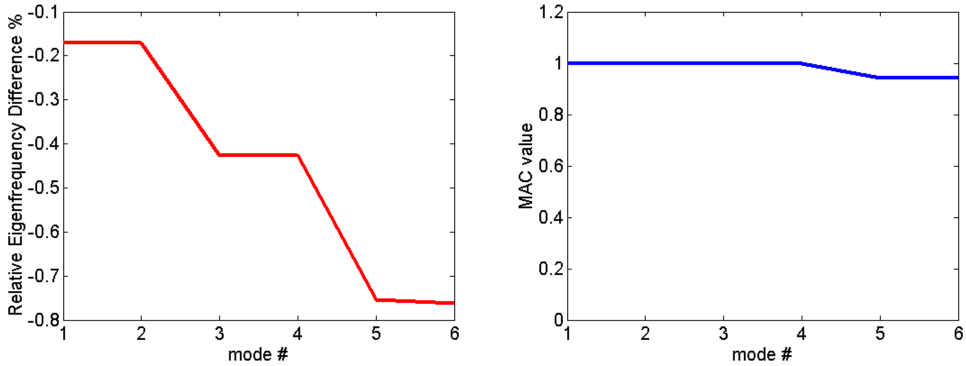


**Figure 3: Comparison of Method A. (original FE model at 100%) with Method C. (Add beams to reduced modal model) in terms of eigenfrequency (left) and MAC value (right). The fundamental modes (1 till 6) are accurately predicted with 0.8% maximum error in eigenfrequency difference.**

# 4 Negative modifications at joints

Complementary to the beam modification analysis tool, a negative modification analysis is applied to the **joint model**. As modification, the joint stiffness (Young's modulus) is considered. Again, three methods can be distinguished.

A. **Modify the original FE model:** create property group for the joint connection and change (decrease) its properties (conventional method).

B. **Add the Guyan superelement with negative properties to the reduced modal model (or to the original FE model)**: Scale the joint superelement to accurately represent the negative modifications in the original model. Add the scaled joint superelement to the reduced modal model (or to the original FE model).

C. **Add joint superelement with positive properties to the modified reduced modal model (or to the original FE model):** modify the original FE model as in Method A. and create the modified reduced modal model from the normal mode solution. Scale the joint superelement and add the scaled superelement to the modified reduced modal model (or to the modified original FE model).

Method A. is the conventional, time-consuming method. Methods B. and C. are more desirable methods for industrial concept modifications: it is quite simple to modify the joint properties in the modal model by using the Nastran card "PARAM, CK2", in which one can specify the multiplication coefficient of the stiffness matrix of the joint. In this Nastran card, it is also allowed to use negative multiplication coefficients, therefore starting from a nominal model at N% of the properties, one can *add* negative stiffness with the aim to decrease the global stiffness properties of the joint.

All three methods have been tested for their capability to predict negative concept modifications to the stiffness (Young's modulus) of joints. Method B. has been compared to Method A. in terms of FRF amplitude. The comparisons are made between the original FE model with shell modification at 100-30% and the reduced modal model at 100% to which Guyan superelement with negative properties at 30% is added. It can be seen [4] that the Guyan superelements are statically equivalent to the modified original FE models, and that also the low and medium frequency range (up to 650 Hz) is accurately predicted. Accurate results are also obtained with Method C.; Figure 4 shows the comparison of Method A. with Method C. The comparison is made between the original FE model at 100% and the reduced modal model at 70% to which a Guyan superelement with positive properties at 30% is added. It can be seen that the reduced modal model is statically equivalent to the modified original FE model, and that also the low and medium frequency range (up to 700 Hz) is accurately predicted. The main interest is that the joint modifications are accurate in the range of global vehicle modes (below 100 Hz).

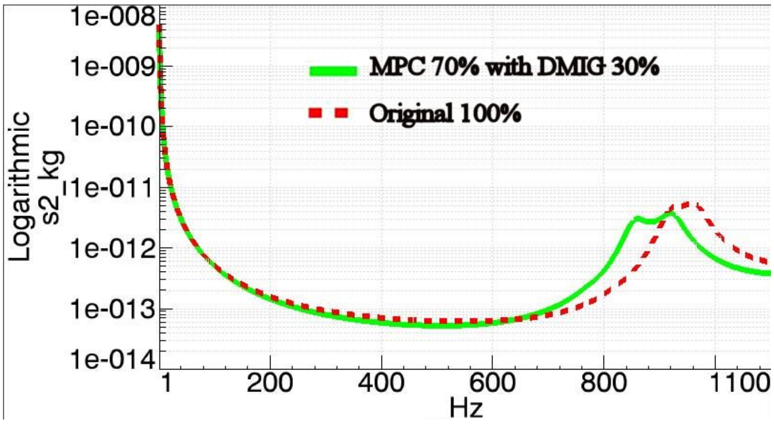


**Figure 4: Comparisons of Method A. (original FE model at 100%) with Method C. (add Guyan superelements to modified reduced modal model). The reduced model is statically equivalent to the modified original FE model.**

## 5 Practical implementation

In this section, positive and negative modifications are applied to the beams and joints in a racing car chassis. The FE model (4391 nodes, 6012 elements) is shown in Figure 5 (left).

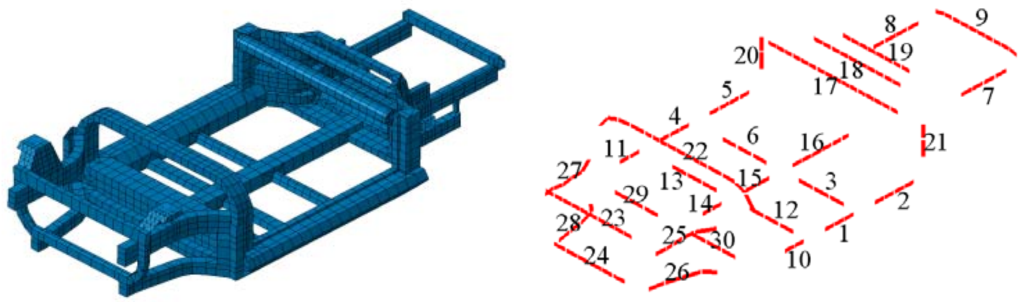

**Figure 5: Racing car chassis: FE model (left) and the 30 equivalent beams computed for the primary beam members (right).**

Using the procedure in Section 2 and in [2], equivalent beams have been computed for all primary beam members of the racing car body. When equivalent beams are computed on symmetric locations in the FE model, it is ensured that also the equivalent beams are symmetric. Thirty equivalent beams have thus been made; see Figure 5 (right). In a beam concept modification framework, one should realize that each design modification typically has a substantial cost involved. Designers should therefore select a limited subset of beams to be modified, while making sure to achieve the design objective. This means that the beams must be selected for which a thickness increase has the highest effect on the objective function. For this purpose, a design sensitivity analysis (DSA) [2] has been performed on the reduced modal model (defined on the wire frame nodes in Figure 7), by iteratively adding 10% equivalent beam thickness to each of the 30 beam locations, and assessing the effect on the objective function “maximize $\mathbf{F}_{\mathrm{sum}}$.”, the sum of the first six natural frequencies. The results are displayed in the DSA coefficient plot in Figure 6.

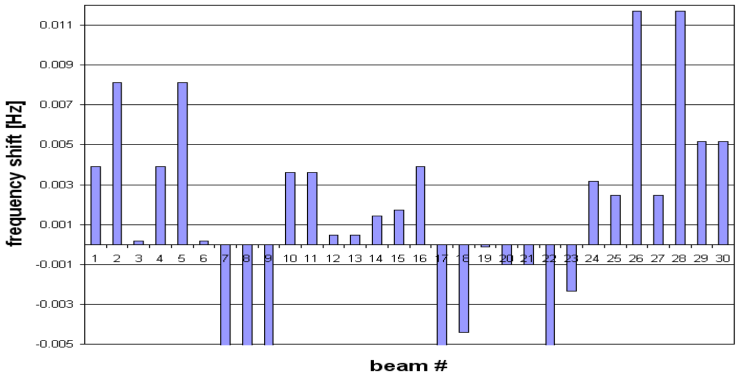


**Figure 6: DSA coefficient bar chart: change of the objective function $F_{sum}$ (vertical) that results from adding 10% equivalent thickness to each of the 30 beam locations (horizontal)**

In [2] positive concept modifications of beams and joints are applied with the aim to maximize $\mathbf{F_{sum}}$. As an example, the beam thickness properties and the joint stiffness properties are modified. The nine most suitable beam locations (with highest positive DSA coefficients, i.e. beam numbers 1, 2, 4, 5, 16, 26, 28, 29, 30, see Figure 6) to increase the beam thickness have been selected. Two additional locations (3, 6) were selected for layout purposes, so that two 3-beam joints (J1 and J2) could be created. The validation of positive beam and joint modifications applied to the reduced modal model at 100% of the nominal properties is performed in [2] using the beam and joint layout in Figure 7; the comparison [2] in terms of FRF amplitude between the modified original FE model and the reduced modal model, when equivalent beams and joints are added with a value of 50% w.r.t. the nominal FE model, clearly validate the accuracy of the proposed method.

On the other side, in a negative concept modification one has to select the beam locations with the lowest negative DSA coefficients (i.e. the highest-magnitude negative values), as decreasing their stiffness will lead to an increase of the objective function. The seven locations (number 7, 8, 9, 17, 18, 22, and 23, see Figure 7) to decrease the beam thickness have been selected from the DSA coefficient plot in Figure 6.

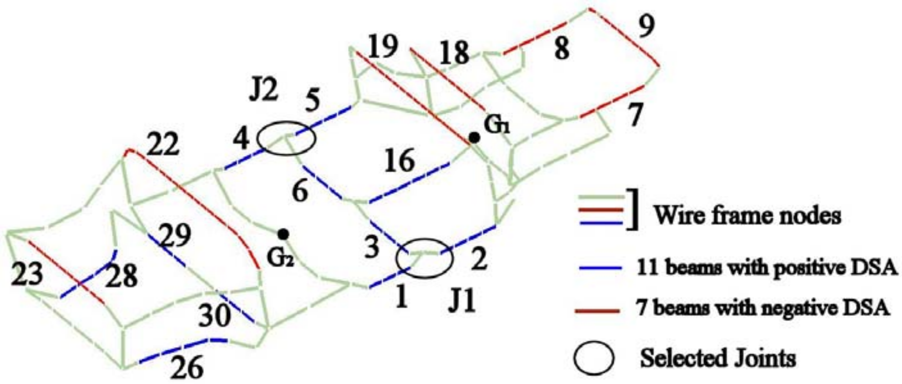


**Figure 7: Reduced modal model on wire frame nodes: in blue, the 9 beam locations with highest positive DSA coefficients and the two selected beams to model the two 3-beam joints; in red, the 7 beam locations with lowest negative DSA coefficients.**

## 5.1 Validation of negative modifications

In this Section the accuracy of negative modifications is verified using the 7 beams with negative DSA coefficient (layout in Figure 7). In analogy with Sections 2 and 3, four methods for a negative beam and joint modification analysis are distinguished:

A. **Modify the original FE model**: Create property groups for beam-like structures and joints in the FE model and change (decrease) the thickness of the beams and/or the Young’s modulus of the joints.

B. **Add beams with negative properties to the reduced modal model**: Scale the equivalent beams to accurately represent the negative modifications in the original model. Add the scaled beams to the reduced modal model of the car.

C. **Add joint model with negative properties to the reduced modal model**: Scale the joint superelement to accurately represent the negative modifications in the original model. Add the scaled joint superelement to the reduced modal model of the car racing chassis.

D. **Add beam model with positive properties to the modified reduced modal model:** modify the original FE model as in Method A and create the modified reduced modal model from the normal mode solution. Scale the equivalent beams and add the scaled beams to the modified reduced modal model of the car chassis.

As before, Method A. is the conventional method, it is inefficient because of the manual interaction that is required to modify the FE model properties, and the necessary full FE model computation to assess the effect of modifications. Method B. is interesting, but one cannot apply negative properties to finite element representations of beams, so that this method is not applicable (more details are given in [4]). Methods C. and D. are applicable and have been performed. Methods C. and D. are efficient methods because the beam and joint modifications are easily applied on a small-sized reduced modal model, for which the structural analysis results are quickly obtained. Therefore, these are desirable methods for industrial concept modifications.

Method A and Method C are compared in terms of FRF accuracy. The modifications involve the two 3-beam joints (J1 and J2) in Figure 7. The comparisons are made between the original FE model with joint modifications at 100-30% and the reduced modal model at 100%, to which joint superelements with negative properties at 30% are added. It can be seen in [4] that the comparisons between the FRF curve of the original model at 100% and FRF curve of the original models at 100-30% show that no substantial concept modifications occur with 30% of joint modifications. Therefore, for this car chassis model, these joint modifications have no influence in the global dynamic performance and the reduced model accurately underlines such behavior. This underlines that the effect of local joint modifications to global vehicle modes is very much application-dependent.

The accuracy of Method D is assessed in the following. Method A. and Method D. are compared in terms of eigenfrequency, MAC values and FRF amplitude. The comparisons are made between the original FE model at 70+30% and the modified reduced modal model at 70% (obtained at joint stiffness properties of 70% w.r.t. the nominal properties) to which the 7 beams (number 7, 8, 9, 17, 18, 22, and 23) with positive properties at 30% are added. It can be seen that the fundamental (first six) modes are accurately predicted by adding equivalent beams to the reduced modal model. Higher modes introduce deformations of the cylinder cross sectional area; these modes are not accurately predicted with equivalent beams. In particular, in the natural frequency comparisons (Figure 8, left), the relative eigenfrequency difference is always less than 3.0%, which confirms the accuracy of Method C. Also in the MAC comparisons (Figure 8, right) the results are good: the diagonal values are 0.98 or higher. The FRF comparison in Figure 9 shows that Method A and Method D overlap up to 50 Hz. For higher frequencies, the FRF amplitude obtained with Method D (on the reduced modal model) are shifted a little downward in the frequency domain.

Overall, it can be said that a maximum error of 3.0% is acceptable in comparison with the advantages that one can achieve using the reduced modal model, namely the little effort in time computation and in the modification procedure. It is the author's opinion that the presented approach can be used for concept CAE purposes: to obtain fast design directions from the results of the concept modifications. The FRF computation times have been compared on a representative PC using Windows XP. Method D. (18s) is much more efficient than Method A. (53s). For more detailed vehicle body models, these reduction factors will be much higher. In the next section, an example optimization study is presented, to illustrate the benefits of the presented procedure in the concept design process.

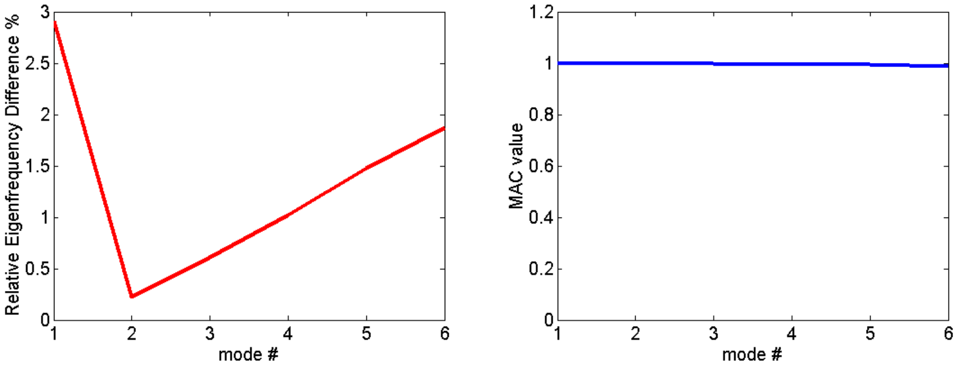


**Figure 8: Comparison of Method A (original FE model at 100%) with Method D (Add beams to reduced modal model) in terms of eigenfrequency (left) and MAC value (right). The fundamental modes (1 till 6) are accurately predicted with 3% maximum error in the relative eigenfrequency difference.**

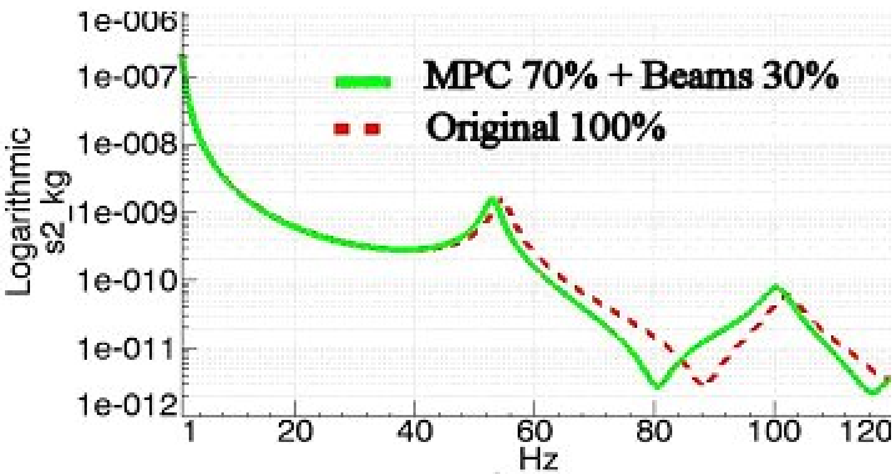


**Figure 9: Comparisons of Method A (original FE model at 100%) with Method D (add beam model to reduced modal model). The two curves overlap up to 50 Hz.**

## 5.2 Optimizations

To further improve the objective function $\mathbf{F}_{\mathbf{sum}}$, an optimization case must be defined [5][6] on the reduced modal model with added beams and joints (See Section 4.1, Method D.). The optimization (CASE 1) has been performed with the seven beams with negative DSA coefficients, the two joints and the eleven beams with positive DSA coefficients. Symmetric beams and joints are modified simultaneously, with allowable ranges between 0 and 80% modification for the 2 joints and 11 beams with positive DSA coefficient, and with allowable ranges between 0 and 30% modification for the 7 beams with negative DSA coefficients. The beam and joint modifications are added to the reduced modal model of the car chassis at 70% of the nominal properties, allowing at the maximum to obtain a model at 150% of the nominal properties in the locations of beams with positive DSA coefficients and at 100% of the nominal properties in the locations of beam with negative DSA coefficient (the accuracy of the 2 joints and the 11 beams with positive DSA coefficients applied to the modified reduced modal model at 70% of the nominal properties in this modification range has been verified in [4]). Comparing the optimized objective values in [2] (obtained using the 11 beams and 2 joints with positive DSA coefficient) and in CASE 1 (Table 1), the objective value of CASE 1 is clearly the highest, therefore the beams with negative DSA coefficient play an important role in the determination of the maximum objective function; more specifically, the optimized objective value with 11 beams and 2 joints in [2] was 525.91Hz, while in CASE 1 it is 530.15 Hz. More details can be found in [4].

| Objective | $f_1$ | $f_2$ | $f_3$ | $f_4$ | $f_5$ | $f_6$ | $F_{sum}$ |
|---|---|---|---|---|---|---|---|
| Nominal | 54.599 | 55.116 | 87.480 | 98.445 | 102.10 | 116.65 | 514.39 |
| Modify 10% and 80% | 55.044 | 57.985 | 89.075 | 99.428 | 104.61 | 116.77 | 522.93 |
| Optimized (CASE 1) | 55.351 | 57.867 | 89.982 | 99.862 | 107.23 | 119.85 | **530.15** |

**Table 1: Objective Results: Nominal (original model at 70% of the nominal properties); Modify 10% and 80%(reduced modal model at 70% with added joints and beams with positive DSA coeff. at 80% and beams with negative DSA coeff. at 10%); After optimization.**

## 6 Conclusions

In dynamic analysis of flexible structures, it is often required to reduce the order of FE models to achieve accurate eigensolutions (of modified designs) in a reasonable amount of CPU time. In this paper, a concept CAE approach to analyze and optimize the global bending and torsional behavior of a vehicle body have been presented. Concept modifications in the body beam-like sections and in the joint connections are analyzed using the reduced modal model. Negative beam and joint modifications as well as positive modifications have been applied to a racing car chassis to improve the fundamental vehicle dynamics. After a sensitivity analysis to find the most suitable modification locations, an optimization case has been defined to determine the size of the modifications. The reduced models of beam and joint accurately reproduced the low-medium frequency behavior of the original models, both with positive modifications (assessed in [2]) and with negative modifications, assessed in this paper. It can be concluded that the presented beam and joint modification approaches are accurate and efficient to obtain concept design directions, when compared to the conventional modifications on full FE models.

## Acknowledgements

The work presented in this paper has been performed in the framework of the Marie Curie Host Fellowship "EDSVS" for early stage researches training. The author would like to thank the European Commission for the grant received. In a wider sense, the research fits in the ongoing research project "Analysis Leads Design – Frontloading Digital Functional Performance Engineering", a joint project between LMS International and K.U.Leuven, Department of Mechanical Engineering, Division PMA. This project is supported by IWT Vlaanderen.

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