Gain-scheduling control of machine tools with varying structural flexibilities

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Abstract

The high accelerations occurring in present-day machine tools are likely to excite the vibration modes of the machine structure. These structural eigenfrequencies are not constant but depend on the position of the tool in its workspace. High performance motion controllers should take into account these varying resonances. This paper discusses the gain-scheduling control approach for an experimental set-up containing a flexible beam of which the stiffness depends on its length. H∞-controllers are designed for several constant beam lengths and are linearly scheduled. Next to this ad-hoc scheduling, analytically scheduled controllers are synthesised. It is shown that the inherent conservatism in the design method limits the performance of the analytical approaches.

1 Introduction

The growing competition on the international markets generates a demand for faster machine tools that can reduce machining time, while preserving or improving the final accuracy. High accelerations of these machines excite the machine structure up to high frequencies thereby exciting the structure's modes of vibration. These structural vibrations need to be damped if accurate positioning and trajectory tracking are to be achieved.

An additional problem is that the dynamic behaviour of the machine tool depends on the position of the tool as a consequence of the varying machine configuration during machining. Such time-varying behaviour cannot be controlled by classical linear control methods, as these methods require a linear time invariant (LTI) model of the system. One solution to this problem is to ensure that the designed LTI-controller makes the system behaviour robustly stable against the varying dynamics of the machine tool. This robust stability requirement, however, normally comes with low performance.

Robust control techniques assume system dynamics lie within an uncertainty band around the nominal value. In machine tools, the dynamical behaviour depends on the position of the tool, which can be measured in real-time. The performance of the controlled system therefore could be improved if this knowledge is included in the controller by making the controller dependent on the instantaneous configuration of the machine tool. In classical gain scheduling, LTI controllers are designed for different values of the varying parameter and scheduled ad-hoc, for example by linear interpolation. Because of this ad-hoc scheduling, extensive simulations or experiments are needed to guarantee robustness of the closed-loop system. More recent design methods for gain scheduling start from an analytical linear parameter varying (LPV) description of the model and criteria for robustness can be derived analytically. In this paper, both ad-hoc and analytical scheduling techniques are applied to an experimental set-up containing a flexible beam of which the eigenfrequency depends on its length.

In section 2 the set-up is described. Section 3 deals with the identification of the set-up. The identification results are used in Section 4 to design different LTI controllers for constant beam lengths. In Section 5 both ad-hoc and analytically scheduled controllers are synthesised for the set-up. Finally some conclusions and directions for further research are highlighted in section 6.

2 Description of the set-up

The control of the X-axis of a Philips 4-axis pick-and-place machine is considered (see Figure 1). The machine consists of a gantry driven by two linear motors controlling the Y-motion. The X-motion of the carriage over the gantry is also driven by a linear motor. The vertical Z-motion is a traditional rotary motor drive with ball screw/nut combination. Rotation of the quill around the Z-axis constitutes the fourth axis. For this analysis only the X-axis linear motor is used, along with the vertical Z-axis motor.

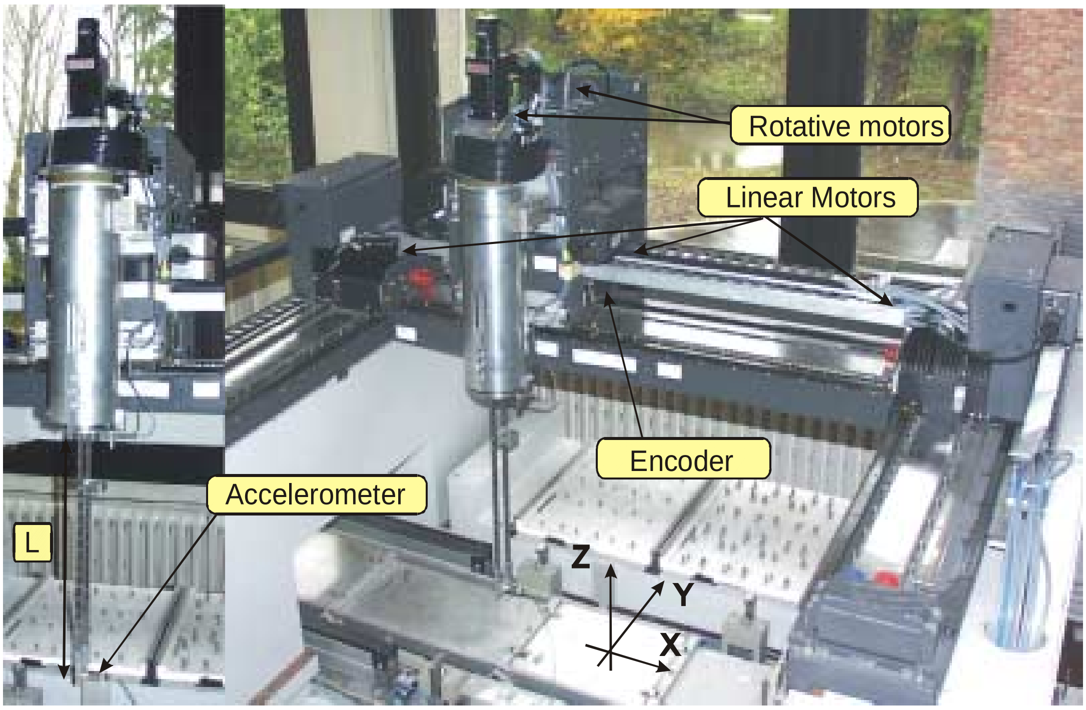


Figure 1: Set-up: pick-and-place machine.

The position of the linear motor and the length of the beam are measured with optical encoders and the acceleration of the end point of the beam, representing the tool tip in this set-up, is measured with an accelerometer.

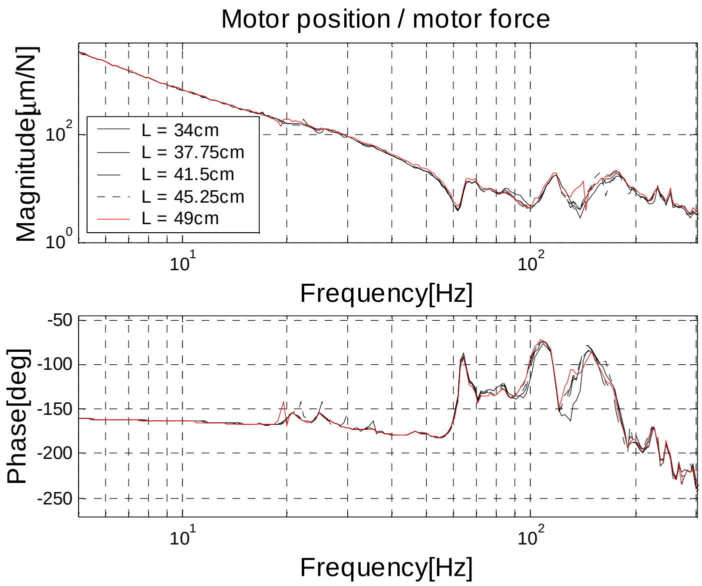
The objective is to move the end point of the beam as accurate and fast as possible along a prescribed trajectory in the X-Z plane. Fast movements of the linear motor will excite the eigenfrequencies of the flexible beam and during motion, the length of the beam is continuously changed, giving rise to varying vibration frequencies.

3 Experimental identification of the set-up

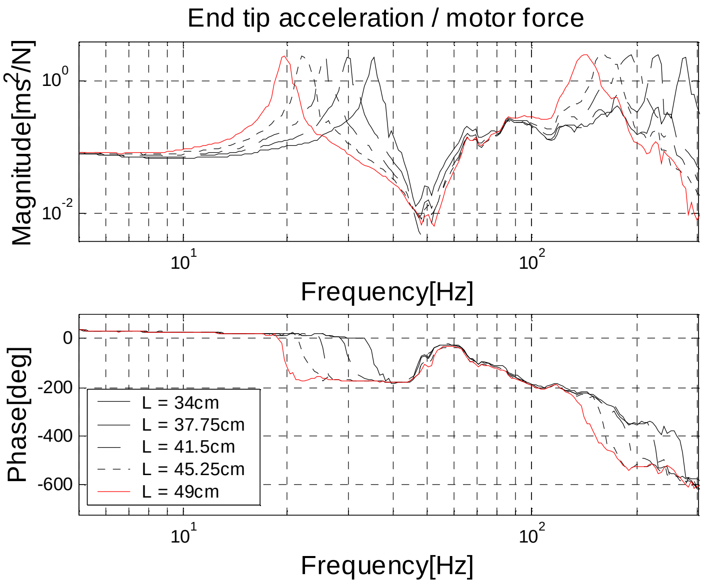
Linear models of a system can be obtained by fitting black-box models on the frequency response functions (FRFs) of the system. Figure 2 shows two FRFs, from the linear motor force excitation (input) to the linear motor position (output) and to the end point acceleration (output), for different lengths of the beam using a stepped-sine input signal.

As can be inferred from Figure 2(a), the position FRFs have a mass-line characteristic for low frequencies and some resonances at higher frequencies. The effect of the length of the beam on these FRFs can be neglected. The accelerometer FRFs on the contrary clearly show a dependency of the first two eigenfrequencies on the length of the beam (see Figure 2(b)).

As one system is considered with one excitation signal and two measurement signals, a single-input-multiple-output (SIMO) model should be fitted to the different FRFs. This is however much harder to realise than fitting two single-input-single-output (SISO) models, and as the influence of the resonance of the flexible beam on the FRF of the motor is negligible, it is justified to model the system as two SISO models. A mass-line is therefore identified on the postion FRFs and dynamic models of order eight are fitte on the accelerometer FRFs (see Figure 3).



(a) FRF for linear motor force excitation (N) to motor position (µm).



(b) FRF for linear motor force excitation (N) to end point acceleration (m/s^2).

Figure 2: FRFs for linear motor force excitation.

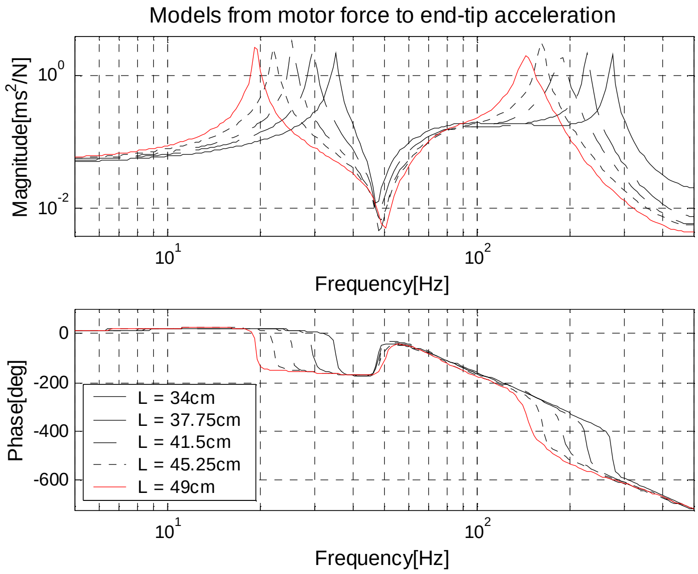


Figure 3: Models fitted on the accelerometer FRFs.

4 LTI control design

4.1 General scheme

The objective of the control design is to move the end point of the beam as accurate and fast as possible. To damp the vibrations of the flexible beam, different control methodologies are possible.

As shown by previous studies [1], an appropriate way to design a motion and vibration controller is to design the motion controller around the system with a vibration controller in a HAC-LAC structure [2]. Figure 4 shows this control scheme in which the high-authority motion controller (HAC) is built around the low-authority vibration controller (LAC).

The motion controller $C_1(s)$ is a standard lead-lag controller [3].

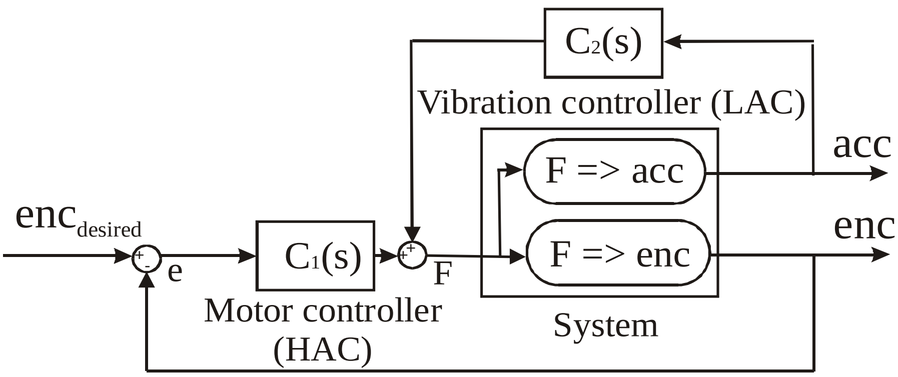


Figure 4: Control scheme.

4.2 Vibration controller for fixed beam lengths.

From now on, the paper concentrates on the vibration controller design that aims at damping the first eigenfrequency of the flexible beam ($C_2(s)$ in Figure 4).

Different controllers have been designed for the vibration control ranging from loop shaping, over pole placement to H_∞-controllers [3,4]. This paper focuses on the H_∞ method as it yields the best performance and is the most versatile to use.

In the H_∞ control framework, a controller is synthesised for a generalised plant that results in a stable closed-loop system with minimal H_∞-norm [5]. Augmenting the original plant with specific weighting functions results in a closed-loop system with the desired properties. Care has to be taken when choosing these weighting functions as they increase the complexity of the controller. For this set-up the models of Figure 3 are used in the control design. An iterative design (see [4]) showed that two weighting functions of order three were sufficient to achieve the desired closed-loop characteristics, leading to H_∞-controllers of order fourteen for the five fixed beam lengths (see Figure 5), which does not pose any problems for present-day signal processing systems.

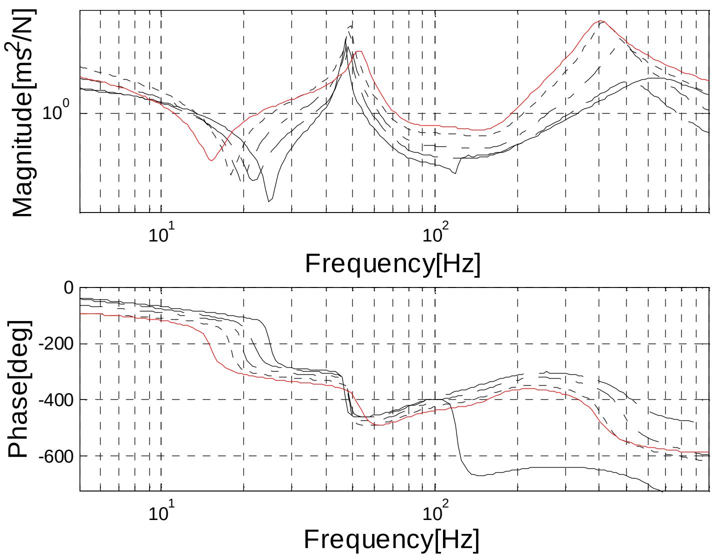


Figure 5: H_∞-controllers ($C_2(s)$) for fixed beam lengths.

4.3 Experimental results

To validate the performance of the controllers a trajectory is imposed that sufficiently excites the first resonance of the beam. A reference trajectory of 400 mm, based on a constant acceleration and deceleration profile of 10 m/s^2 and a constant velocity of 1 m/s, is chosen.

Figure 6 shows the damping effect of the H_∞ vibration controller for the shortest beam length (L=34cm). Similar results are obtained for the other beam lengths. Experiments showed that a controller designed for a certain beam length has low performance or even leads to instability when it is used for one of the other design lengths. A controller that is stable for all beam lengths has very low performance, as will be shown in the next section.

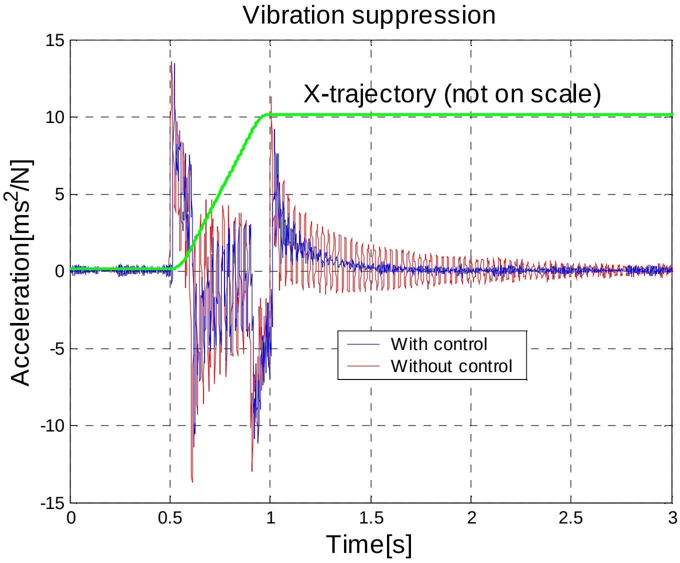


Figure 6: Performance of the H∞-controller for the shortest beam length.

5 Gain-scheduling

A robust H∞-controller has been designed for the set-up where the variation of the beam length is modelled as a parametric uncertainty. Figure 7 shows the performance of this controller that is stable for all beam lengths for a test trajectory during which the length of the beam is varied with a constant speed. Only a little increase in damping is visible for the large beam length.

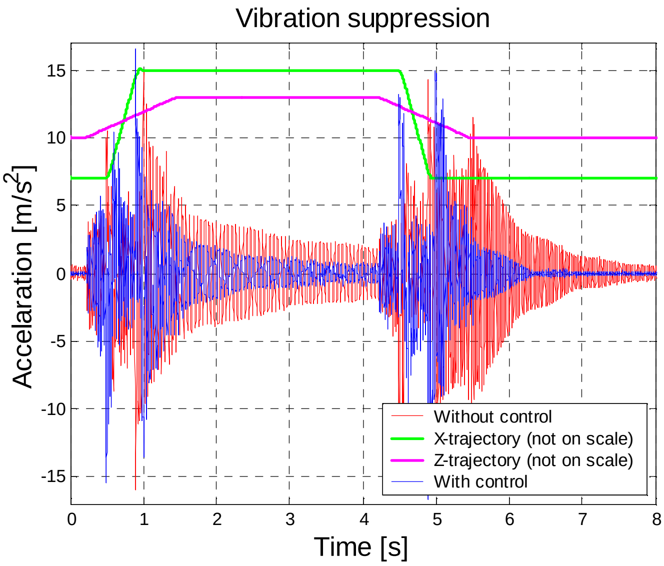


Figure 7: Performance of the robust H∞-controller.

To increase performance, in this section the controller is scheduled, based on the beam length. First ad-hoc linear scheduling of the designed H∞-controllers is used and thereafter analytical scheduling techniques are proposed.

5.1 Ad-hoc linear scheduling

A scheduling technique that has a long history in control design subdivides the working space into different working regions. For these regions an LTI model is identified and a controller synthesised. These controllers are then combined in a scheduled controller. The outputs of the different controllers can be combined using fuzzy laws, or the parameters of the controllers themselves can be scheduled.

The idea to schedule the H_∞-controllers is to vary the parameters of the numerator and denominator of the controller in transfer function form according to the beam length. This dependency is however very nonlinear for several coefficients, and the transfer function description of a system is numerically ill-conditioned. Closer analysis of the pole-zero map of the different controllers shows that these poles and zeros vary approximately linearly as a function of the beam length. After slight modification the H_∞-controllers can be efficiently linearly scheduled based on the varying poles and zeros.

Figure 8 shows a significant increase in damping with the scheduled H_∞-controller compared to the robust controller of Figure 7. The performance of this ad-hoc scheduled controller is only slightly smaller than the performance of the LTI controllers. Although the performance of the scheduled controller is high for all tested trajectories it cannot be guaranteed theoretically.

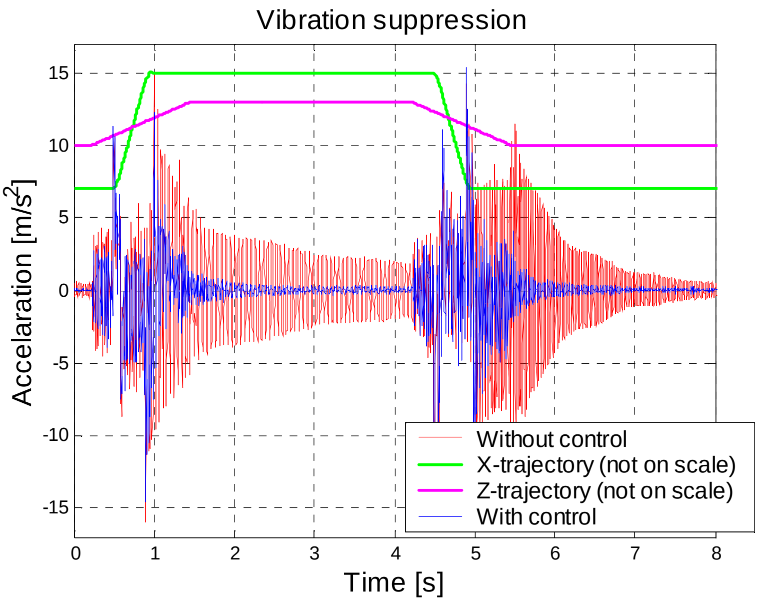


Figure 8: Performance of the linearly scheduled H_∞-controller.

5.2 Analytic LPV controllers

To avoid extensive experiments needed to guarantee performance and robustness of ad-hoc scheduled controllers, analytically scheduled controllers can be designed. The starting point then is an LPV model:

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{A}(\mathrm{L})\mathbf{x} + \mathbf{B}(\mathrm{L})u \\ y &= \mathbf{C}(\mathrm{L})\mathbf{x} + \mathbf{D}(\mathrm{L})u, \end{aligned} \tag{1}$$

from which an LPV controller can be synthesised. Control design methodologies for LPV models are available for system matrices that depend in an affine way on functions of the parameter L [6], or in a fractional way on L [7]. Both methods are presented hereunder.

5.2.1 Affine parameter dependent model

By analogy with affine functions, which are defined as the sum of a constant term and a linear function, an affine parameter dependent model has the form of Equation 1 with:

$$\mathbf{A}(\mathrm{L}) = \mathbf{A}_0 + p_1(\mathrm{L})\mathbf{A}_1 + \cdots + p_\mathrm{n}(\mathrm{L})\mathbf{A}_\mathrm{n} \ , \tag{2}$$

where $\boldsymbol{A}_\mathrm{i}$, i=0,...,n are square matrices and p_i(L), i=1,…,n arbitrary functions of L. Similar expressions are valid for $\boldsymbol{B}$(L), $\boldsymbol{C}$(L) and $\boldsymbol{D}$(L). A general identification method to obtain an affine parameter dependent model for an experimental set-up is not available yet. In this paper, an ad-hoc identification scheme is used that starts from the pole-zero description of the system for fixed beam lengths. By element-wise conversion of these poles and zeros to a state-space formulation, a global parameter dependent model is obtained that fits the individual models with small error. Notice that this procedure starts from identified LTI models and does not guarantee that the model is a correct representation of reality for varying beam lengths.

The control design for such an affine model is based on extended concepts of H_∞ control theory [6] and the resulting problem is formulated as a linear matrix inequality (LMI). A single quadratic Lyapunov function is looked for to guarantee stability over the global parameter range and the resulting controller depends in the same way on the beam length as the system according to the functions p_i(L), i=1,…,n in Equation (2). The resulting controller has a very low performance compared to the ad-hoc scheduled H_∞-controller. The reason lies in the design of the quadratic Lyapunov function. This function guarantees stability for infinitely fast varying parameters. In reality the speed of variation is limited and looking for one single Lyapunov function introduces considerable conservatism in the design. This conservatism can be reduced if a parameter-dependent Lyapunov function is searched for. The size of the resulting LMI to synthesise such a controller in general however grows above the limits of present-day solvers and is as such not applicable to practical problems.

5.2.2 Model in linear fractional form

The controller in the former section depends on the length of the beam in an affine manner following Equation 2. Affine functions are not suited to capture highly nonlinear variations. A more extended technique is offered by the Linear Fractional Transformation (LFT) form where the parameter dependency is allowed to be fractional, as a ratio of two polynomials. This form is extensively used in robust control theory and is defined as (see Figure 9):

$$o = [\mathbf{M}_{11} + \mathbf{M}_{12}\Delta(\mathrm{L})(\mathbf{I} - \mathbf{M}_{22}\Delta(\mathrm{L})^{-1})\mathbf{M}_{21}]i \ , \tag{3}$$

with i the input and o the output of the system. The parameter dependency is extracted from the model description and placed in a feedback loop. A general approach to derive an LFT model for a practical set-up is also not yet available in literature, as was the case for an affine model. For the set-up considered in this paper, a similar strategy as for the affine model, based on poles and zeros, is followed. The resulting $\Delta(\mathrm{L}) = \mathrm{L}\boldsymbol{I}_{9\mathrm{x}9}$ with $\boldsymbol{I}$ the identity matrix. The size of the block is determined by the number of poles of the different LTI systems.

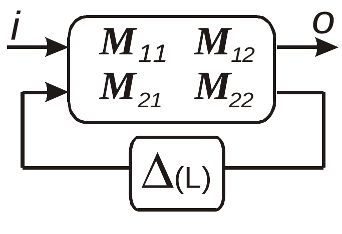


Figure 9: LFT scheme.

The design of LPV controllers for LFT systems is also based on extended H_∞ control concepts but is more complicated than the control design for affine models, as a larger LMI problem has to be solved [7]. This rise in complexity comes with the advantage that the controller does not have to depend on the parameter in the same way as the plant. The performance of the resulting controller is again very low. The reason for this low performance is that also the LFT control methodology searches for a single quadratic Lyapunov function, which again introduces considerable conservatism in the design.

5.2.3 Discussion and further research

Although they have guaranteed robustness, both parameter dependent model structures that were used for analytical LPV synthesis resulted in controllers with very low performance compared to the ad-hoc scheduled H_∞-controllers. The performance is even lower than that of the robust controller shown in Figure 7 and for reasons of brevity the experimental results with these controllers are omitted.

A parameter dependent Lyapunov function for the control synthesis cannot be used as this blows up the system of LMIs that needs to be solved. For stability analysis the size of the resulting LMI is still feasible. Such a stability analysis is performed for the closed loop system with an affine model of the ad-hoc scheduled H_∞-controller and using an affine parameter-dependent Lyapunov function [8]. This analysis however could not guarantee stability of the system, even for constant beam lengths, although the poles of the closed loop system for all parameter values were negative. This proves that even an affine parameter-dependent Lyapunov function is too restrictive for the control synthesis for the set-up.

For the moment, it is not clear if the same degree of conservatism will be introduced for machines tools with more damped modes. Further research is necessary to get more insight in the physical meaning of the quadratic Lyapunov function in the LPV design. Another interesting idea to investigate is whether it is possible to extend the ad-hoc scheduled controller with a term that incorporates the derivate of the varying stiffness to guarantee stability for dynamic systems with fast varying parameters. The idea is than that the controller gain decreases for rapidly varying parameters, hereby insuring the stability of the system, while high controller gains are used for slowly varying parameters. This way the high performance of the ad-hoc scheduling could be combined with a guaranteed stability for dynamic systems with fast varying parameters. Systematic methods to achieve such scheduling will be look for in the future.

6 Conclusion

In this paper the experimental implementation of (linear) gain-scheduling vibration controllers for a machine tool with varying dynamic stiffness has been presented. Scheduling the controller results in a significant increase in performance compared to an LTI controller that is robustly stable for the system variations.

Both ad-hoc and analytical scheduling have been implemented. The study shows that for motion and vibration control of machine tools with varying flexibilities high performance can be obtained with ad-hoc linear scheduling but further improvement of the LPV-methods is necessary to be able to design high-performance analytically scheduled LPV controllers.

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8 References

[1] H. Van Brussel, W. Symens, *Vibration and Motion Control of Mechatronic Systems*, CD-Rom *Proceedings of the Acomen conference 2002*.

[2] Preumont, A., *Vibration Control of Active Structures*, Kluwer Academic Publishers, Dordrecht (1997).

[3] Symens, W., Swevers, J., Van Brussel, H., *Pole-placement vs. Loopshaping Design for Gain-scheduling Control of Machine Tools with Position Dependent Dynamics*, *CD-Rom Proceedings of the European Control Conference 2003.*

[4] Symens, W., Van Brussel, H., Swevers, J., *Analysis of H_∞ Control Techniques for the Control of Machine Tools with Position Dependent Dynamics*, *CD-Rom Proceedings of the 6th National congress on Theoretical and Applied Mechanics 2003*.

[5] Skogestad, S., Postlethwaite, I.7, *Multivariable Feedback Control: Analysis and Design*, John Wiley & Sons (1997).

[6] Apkarian, P., Gahinet, P., *A Convex Characterization of Parameter Dependent H∞ controllers*, IEEE Transactions On Automatic Control (1995), Vol. 40, No. 5, pp. 853-864.

[7] Scherer, C., 2001, LPV Control and Full Block Multipliers, Automatica, Vol. 37, No. 3, pp. 361:375.

[8] P. Gahinet, P. Apkarian, M. Chilali, M., *Parameter-Dependent Lyapunov Functions for Real Parametric Uncertainty*, IEEE Transactions on Automatic Control (1996), Vol. 41, No. 4, pp. 436-442.