Decoupling feedback control for improved multivariable vibration test rig tracking

D. Vaes*, **W. Souverijns***, **J. De Cuyper**†, **J. Swevers***, **P. Sas***
*KULeuven, Department of Mechanical Engineering,
Celestijnenlaan 300 B, B-3001, Heverlee, Belgium
† LMS International,
Researchpark Haasrode Z1, Interleuvenlaan 68, B-3001 Leuven, Belgium
e-mail: david.vaes@mech.kuleuven.ac.be

Abstract

Within the framework of the tracking problem on an automotive vibration test rig, a new control strategy is developed which is applicable to square multiple-input-multiple-output systems (MIMO-systems) with a certain degree of symmetry. Classical MIMO-controller design is a three step procedure: response measurement, MIMO-identification and MIMO-controller design. The latter two steps are often very cumbersome. In this paper, a new procedure is proposed which avoids these steps. It consists of the following steps: response measurement, almost decoupling of the MIMO-system into multiple single-input-single-output systems (SISO-systems) by transformations of the inputs and the outputs, SISO-identification of the decoupled systems, and multiple SISO-controller design. The great advantage of the decoupled model, is that it allows the use of well-understood, robust SISO-controller design methods. This paper shows that the automotive vibration test rig can be described with a decoupled model, at least as accurate as with a full MIMO-model.

1 Introduction

During the design of a new car, vibration tests on a hydraulic test rig (four poster) are important to adjust comfort and durability properties of the new prototype. An example of such a test rig is shown in figure 1. To make the vibration tests representative for the further life-time of the vehicle, reference signals (accelerations or forces) are measured during a test drive on a special test track. These reference signals (called targets) are reproduced on the test rig.

Figure 1: Vibration test rig.

The calculation of the control signals, for the hydraulic actuators of the vibration test rig, such that the measured signals on the test rig match the target signals, is a multivariable tracking problem. Current industry practice to solve this multivariable tracking problem is to use an off-line iterative process [1]. In [2] it is shown that, extending the current process with a high-performance multivariable feedback controller, allows to reduce the number of iterations significantly. The higher the performance of the feedback controller, the stronger the reduction in number of iterations.

Mostly, the measured target signals are the accelerations of the chassis near the suspension of each wheel. So, from a controller point of view, the four poster can be seen as a system with four inputs (the control signals to the hydraulic actuators of the test setup) and four outputs (the measured accelerations).

The design of a multivariable feedback controller suffers from two major problems. The first one is the identification step. The estimation of an accurate state-space model, which matches well with the measured response, is very difficult. This paper considers frequency domain identification, using the measured FRF matrix of the system. Finding the optimal multiple-input-multiple-output (MIMO) model, based on the measured FRF, is still an important research topic ([3], [4]).

The second problem is the controller design. Most robust control techniques are well-suited for SISOproblems, but are, in practice, hardly usable for MIMO-systems. Especially, modern control methods which can deal with nonlinearities and uncertainties (e.g. Sliding Mode Control), are hardly applicable for MIMO-systems. The four poster and the car form a highly nonlinear system. Particularly in durability tests, when high excitation levels are applied to the car, the suspension behaves extremely nonlinear. The use of robust SISO-control methods to control the MIMO-system would be a great advantage.

The control design procedure proposed in this paper tackles both problems. In fact, the proposed procedure is a new identification procedure, which tries to write the MIMO-system as a combination of p SISO-systems (with p the number of inputs). The described method tries to find the optimal transformation (with constant, frequency independent transformation matrices) of inputs and outputs, in such a way that the relation between the transformed inputs and outputs is decoupled. This means that every input affects only one output. The multivariable controller can then be designed as a combination of robust SISO-controllers between the transformed inputs and outputs.

The present-day state-of-the-art controller for vibration test rigs [5], consists of p independent SISO-controllers, without using a decoupling transformation. In the control design, the influences of other inputs are seen as disturbances which have to be rejected. The decoupling transformation gives the possibility to introduce the interaction of the MIMO-system in the control design, without losing the advantage of designing SISO-controllers.

The paper is organized as follows: the next section starts with a clear description of the decoupling control design procedure. Methods to validate the accuracy of the decoupling transformation are described in the remainder of that section. The first part of the calculation of the transformation matrices is based on Dyadic Transfer function Matrices (DTM), which are described in section 3.1. The DTM-method, which is based on a MIMO-model of the system, and which is only suitable for a very small set of highly symmetrical systems, is adapted for use in this application in section 3.2. Section 4 discusses the practical procedure to calculate the transformation matrices. This is in fact an optimization procedure, so an initial estimate (section 4.1) and a minimization cost function (section 4.2) are required. Then the proposed method is applied to two examples: a Simulink-model of a full car, and a measured FRF of a real four poster.

2 Outline of the proposed control design procedure

2.1 Description of the problem

Throughout this paper the following nomenclature is used: $\tilde{\mathbf{G}}(f)$ is the measured MIMO-FRF matrix at the frequency f, $\mathbf{G}(s)$ is a full MIMO-model of the system, $\mathbf{T_Y}$ and $\mathbf{T_U}$ are the transformation matrices of the outputs (y) and the inputs (u) and $\tilde{\mathbf{D}}(f)$ is the transformation of the measured FRF ($\tilde{\mathbf{D}}(f) = \mathbf{T_Y^{-1}}\tilde{\mathbf{G}}(f)\mathbf{T_U^{-1}}$). $\tilde{\mathbf{D}}_\mathbf{d}(f)$ is defined as $diag\{\tilde{\mathbf{D}}_{ii}(f)\}$ ($diag\{\ldots\}$ denotes the diagonal matrix consisting of the elements between braces). When the decoupling is perfect, the off-diagonal elements of $\tilde{\mathbf{D}}(f)$ are zero, and so $\tilde{\mathbf{D}}(f)$ would be equal to $\tilde{\mathbf{D}}_\mathbf{d}(f)$. In reality decoupling is seldom perfect, and in that case $\tilde{\mathbf{D}}_\mathbf{d}(f)$ is an approximation of $\tilde{\mathbf{D}}(f)$. Finally, $\mathbf{D_d}(s)$ is a diagonal MIMO-model consisting of p independent SISO-models: $\mathbf{D_d}(s) = diag\left\{d_{d_1}(s), \ldots, d_{d_p}(s)\right\}$. So $\mathbf{G}(s)$ is the result of a MIMO-identification of $\tilde{\mathbf{G}}(f)$, and $\mathbf{D_d}(s)$ is the result of p independent SISO-identifications of the diagonal elements of $\tilde{\mathbf{D}}_\mathbf{d}(f)$.

The goal of the decoupling transformation is to find an accurate description of the system in the following form:

$$\mathbf{Y} = \mathbf{T_Y}\mathbf{D_d}(s)\mathbf{T_U}\mathbf{U}. \tag{1}$$

The relation between the transformed inputs $\mathbf{U}' = \mathbf{T_U}\mathbf{U}$ and the transformed outputs $\mathbf{Y}' = \mathbf{T_Y^{-1}}\mathbf{Y}$ is decoupled because:

$$\mathbf{Y}' = \mathbf{D_d}(s)\mathbf{U}' = diag\left\{d_{d_1}(s), \ldots, d_{d_p}(s)\right\}\mathbf{U}'. \tag{2}$$

Each transformed input affects only one transformed output. So the MIMO-controller can be designed as p SISO-controllers between these transformed inputs and outputs. Figure 2 shows the proposed control scheme. The SISO-controllers $k_i(s)$ are based on the SISO-systems $d_{d_i}(s)$. Notice that also the target signals $\mathbf{r}$, must be transformed by the output transformation matrix $\mathbf{T_Y}$.

The calculation of the transformation matrices in section 4 is based on the measured FRF. So, no MIMO-identification must be performed. Once the transformation matrices are determined, the diagonalized FRF ($\tilde{\mathbf{D}}_\mathbf{d}(f)$) can be calculated at every frequency where the original FRF is measured. The next step is to identify the diagonal elements of $\tilde{\mathbf{D}}_\mathbf{d}(f)$. This step consist of p SISO-identification procedures. Remark that in practical situations p SISO-identifications take less time than one $p \times p$

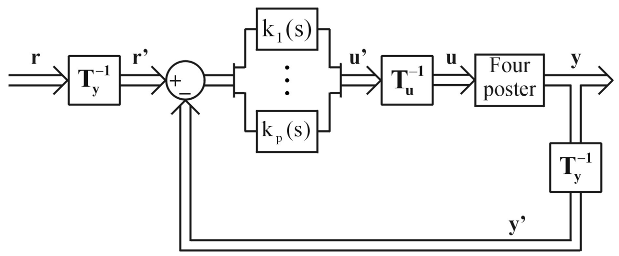

Figure 2: Proposed control scheme: decoupling transformation of inputs and outputs, and p independent SISO-controllers.

MIMO-identification. Based on these identified models, p SISO-controllers can be designed between the transformed inputs and transformed outputs.

2.2 Validation of the results

A first way to check the quality of the decoupling transformation, is to calculate the interaction which remains in the diagonalized system. In [6] some interaction indicators are explained. The easiest-to-use interaction indicator is the so-called *interaction index* $\lambda(f)$. The *interaction index* of a matrix $\mathbf{Z} \in \mathbb{C}^{p \times p}$ is defined as the maximum eigenvalue (the Perron root) of the matrix $\mathbf{C}_+\mathbf{D}_+^{-1}$ where $\mathbf{D}_+ = diag\{|z_{11}|, |z_{22}|, ..., |z_{pp}|\}$ and $\mathbf{C}_+$ the matrix of the absolute value of the off-diagonal elements.

If the interaction index is less than one, the MIMO-system is called *generalized row diagonal dominant*. This means that one output is mainly determined by its corresponding input, and the influence of the other inputs is rather small. More formally stated: A matrix $\mathbf{Z} \in \mathbb{C}^{p \times p}$ is *generalized row diagonal dominant* if there exists an $\mathbf{x} \in \mathbb{R}^p$, $\mathbf{x} > 0$ such that:

$$|z_{ii}|\, x_i > \sum_{j=1,\, j\neq i}^{p} |z_{ij}|\, x_j \quad \text{for} \quad i = 1, 2, \ldots, p. \tag{3}$$

As mentioned before, the transformation matrices will be calculated based on the measured FRF. The decoupling will be sufficient at those frequencies where the interaction index of $\tilde{\mathbf{D}}(f)$ is less than one. This decoupled FRF can be controlled within this frequency band, by independent SISO-controllers which are based on identified models of the diagonal elements of the decoupled FRF.

A second way to check the quality of the decoupling transformation, is based on the goal of the decoupling transformation, namely to find the optimal description of the MIMO-system in the form of equation (1). A comparison of the measured FRF and $\mathbf{T_Y D_d}(s)\mathbf{T_U}$ gives a good idea of the decoupling quality.

When discussing the results in section 5, the correspondence of $\mathbf{T_Y D_d}(s)\mathbf{T_U}$ and the measured FRF is compared with the correspondence of a MIMO-identified model $\mathbf{G}(s)$ and the measured FRF. In practice, no MIMO-model is available. As mentioned before, it can be cumbersome to identify one. But in this paper, the effort to identify a MIMO-model is made, because this results in an easy quality check of the decoupling transformation. When the decoupled description fits as well as an optimal MIMO-model with the measured FRF, the decoupling has succeeded. This means that the MIMO-controller can be designed as p independent SISO-controllers based on a model with the same accuracy as a classical MIMO-model. So, the advantages of robust, well-understood SISO-control can be fully applied to MIMO-systems. In practice, the decoupled description can be compared with the measured FRF, in the same way as a MIMO-model and the measured FRF are compared in a classical frequency domain identification procedure.

It is clear that both ways to check the quality of decoupling boil down to the same. When the interaction index is small, the off-diagonal elements of $\tilde{\mathbf{D}}(f)$ will be small, and so the errors which are made, when $\tilde{\mathbf{D}}(f)$ is approximated by the identification of its diagonal elements in $\mathbf{D_d}(s)$, will be small too. This will be shown in section 5. The second way to check the quality of the decoupling transformation is not only a more natural way to think about decoupling within the framework of this paper, but the comparison with the MIMO-model gives also a satisfaction-level to decide if the decoupling is well enough.

So the goal of the decoupling transformation is to find an accurate description of the system in the form of equation (1), and this goal is reached if this decoupled description describes the system at least as accurate as an optimal MIMO-identified model $\mathbf{G}(s)$.

3 Decoupling based on DTM

3.1 Dyadic Transfer function Matrices

Reducing MIMO-controller design to some SISO-designs is often advantageous. Therefore, many methods have been developed to apply SISO-designs to MIMO-plants. A first class of methods tries to obtain decoupling via high-gain feedback ([7], [6]). Those methods suffer from two major problems. First of all it is very difficult to find high gain feedback controllers which decouple the system and don't destabilize the system. Secondly these controllers are based on a MIMO-model of the system. One of the advantages of the decoupling proposed in this paper is that difficult MIMO-identification is no longer necessary.

The easiest-to-use class of decoupling methods is based on matrix-decompositions. MacFarlane [8] introduces in 1970 the so-called *commutative controller*, based on an eigenvalue decomposition. In 1982 Hung [9] launched a controller based on the singular value decomposition. The decomposition as proposed in equation (1), puts no constraints on the transformation matrices $\mathbf{T_U}$ and $\mathbf{T_Y}$, except that they must be invertible to be used in the control scheme of figure 2. If eigenvalue or singular value decompositions are used, some constraints are inherent in the used method: in the eigenvalue decomposition the left and right transformation matrices are the inverse of each other, in the singular value decomposition the left and right transformation matrices are unitary matrices.

A general decoupling transformation is described in the theory of the *Dyadic Transfer function Matrices* (DTM). A $p \times p$ transfer function matrix $\mathbf{G}(s)$ is called dyadic if there exist constant $p \times p$ matrices $\mathbf{T_U}$ and $\mathbf{T_Y}$ and rational transfer functions $g_1(s), \ldots, g_p(s)$ such that:

$$\mathbf{G}(s) = \mathbf{T_Y} diag\left\{g_1(s), \ldots, g_p(s)\right\}\mathbf{T_U}. \quad (4)$$

Owens [10] developed a procedure to determine transformation matrices $\mathbf{T_Y}$ and $\mathbf{T_U}$, which decouple the system $\mathbf{G}(s)$ if it is dyadic. First two different constants c_1 and c_2 must be chosen. Then the columns of $\mathbf{T_Y}$ can be calculated as the eigenvectors of $\mathbf{G}(c_2)\mathbf{G}^{-1}(c_1)$ and the columns of $\mathbf{T_U}^{-1}$ as the eigenvectors of $\mathbf{G}^{-1}(c_1)\mathbf{G}(c_2)$.

If a system is dyadic, the transformation matrices found with the described procedure are real and constant. If the transformation found with this procedure is not real, the system is not dyadic. In that case, the transformation matrices don't decouple the system perfectly. The choice of the constants c_1 and c_2 is completely free as long as $\mathbf{G}^{-1}(c_1)$ and $\mathbf{G}^{-1}(c_2)$ exist. A natural way of choosing them, is choosing two frequencies f_1 and f_2 yielding $c_1 = j2\pi f_1$ and $c_2 = j2\pi f_2$.

The fact that a dyadic system can be decoupled by the matrices defined in the procedure of Owens can easily be understood, because then the following two equations hold:

$$\mathbf{G}(c_1) = \mathbf{T_Y} diag\left\{g_1(c_1), \ldots, g_p(c_1)\right\}\mathbf{T_U}, \quad (5)$$

and

$$\mathbf{G}(c_2) = \mathbf{T_Y} diag\left\{g_1(c_2), \ldots, g_p(c_2)\right\}\mathbf{T_U}. \quad (6)$$

Now it is easy to calculate the following equation:

$$\mathbf{G}(c_2)\mathbf{G}^{-1}(c_1)\mathbf{T_Y} = \mathbf{T_Y} diag\left\{\frac{g_1(c_2)}{g_1(c_1)}, \ldots, \frac{g_p(c_2)}{g_p(c_1)}\right\}, \quad (7)$$

which shows that $\mathbf{T_Y}$ is found as the eigenvector of $\mathbf{G}(c_2)\mathbf{G}^{-1}(c_1)$. In the same way, it can be shown that the eigenvectors of $\mathbf{G}^{-1}(c_1)\mathbf{G}(c_2)$ determine the transformation matrix of the inputs $\mathbf{T_U}^{-1}$.

In [11] it is shown that for a system which is dyadic, a feedback controller, as in figure 2, is internally stable if and only if, all transfer functions $(1 + d_{d_i}(s)k_i(s))^{-1}$ are stable and there are no right half plane pole-zero cancellations between $d_{d_i}(s)$ and $k_i(s)$. This means that the SISO-controllers can be designed really independently, without concerning MIMO-stability.

It is clear that only a few systems are perfectly dyadic. The symmetry in the system must be very large. In [11], the DTM-method is used to control the active suspension of an horizontal spray boom. The actuators of the active suspension are two identical hydraulic actuators, symmetrically placed around the middle of the spray boom. Because the spray boom itself is also symmetrical, the two excitations can be transformed to a translational and rotational excitation which are independent of each other. Remark that the DTM-property is lost when the actuators are not identical.

The symmetry in the four poster is not so distinct. Remember that the inputs are the control signals to the hydraulic actuators, and the outputs are the measured accelerations of the chassis near each wheel. Assume that the inputs and the outputs are numbered as follows: 1 is front left, 2 is front right, 3 is rear left and 4 is rear right. Using the symmetry in the four poster (e.g. the influence of input 1 on output 1 will be almost the same as the influence of input 2 on output 2 because the suspensions on left and right are the same), the system can be described as follows:

$$\mathbf{G}(s) = \begin{bmatrix} g_{11}(s) & g_{12}(s) & g_{13}(s) & g_{14}(s) \\ g_{12}(s) & g_{11}(s) & g_{14}(s) & g_{13}(s) \\ g_{31}(s) & g_{32}(s) & g_{33}(s) & g_{34}(s) \\ g_{32}(s) & g_{31}(s) & g_{34}(s) & g_{33}(s) \end{bmatrix}. \quad (8)$$

The symmetry is not complete, because the suspension of the front-wheels differs from the suspension of the back-wheels. Nevertheless, the system can be described by 8 transfer functions instead of 16. So there is some symmetry in the system, but not enough for the system to be dyadic. From practical measurements it can be further concluded that the coupling between opposite corners is rather small, so g_{14} and g_{23} are small. Because the system shows a high level of symmetry and has some level of diagonal dominance by itself, it could be assumed, that a decoupled description is accurate, although the system is not dyadic. This was the main motivation to look for decoupling transformations for this type of system.

The DTM-procedure to calculate the transformation matrices, as described by Owens still starts from a MIMO-model of the system and is only applicable if the system is perfectly dyadic. Because avoiding MIMO-identification is one of the main goals of the decoupling transformation, and because the four poster is not perfectly dyadic, this DTM-procedure is adapted for application on the considered system in the next section.

3.2 Adaptation of DTM for decoupling based on measured FRF's

This section applies Owens' method to a measured FRF. In order to do that, three major changes to the original procedure are necessary. First of all, the measured FRF is not known at every arbitrary frequency. The number of frequency points where the FRF is known depends on the number of samples per period of the excitation signals, and the minimal and maximal frequency which is excited. So, c_1 and c_2 cannot be chosen arbitrarily, but two frequencies f_1 and f_2 must be chosen where the FRF is measured. Then the transformation matrices can be calculated as the eigenvectors of $\tilde{\mathbf{G}}(f_2)\tilde{\mathbf{G}}^{-1}(f_1)$ for $\mathbf{T_Y}$ and the eigenvectors of $\tilde{\mathbf{G}}^{-1}(f_1)\tilde{\mathbf{G}}(f_2)$ for $\mathbf{T_U^{-1}}$.

A second problem encountered when applying Owens' procedure to a system which is not dyadic, is the fact that the calculated transformation matrices become complex. If these complex transformation matrices are used to decouple the system, the system is perfectly decoupled at the frequencies f_1 and f_2, but not at other frequencies: the off-diagonal elements of the decoupled FRF ($\tilde{\mathbf{D}}(f) = \mathbf{T_Y^{-1}}\tilde{\mathbf{G}}(f)\mathbf{T_U^{-1}}$) are zero at f_1 and f_2, but not at other frequencies. Those off-diagonal elements are however small at the other frequencies, if the system is nearly dyadic.

Complex transformation matrices are not usable. Control signals as well as measured outputs have real values. Many possibilities to avoid the complex transformation matrices have been investigated, and it appears that retaining the real part of the complex transformation matrices results in the best transformation (better than using the absolute value). The quality of the transformation is measured with the tools of section 2.2. Due to neglecting the imaginary part of the transformation matrices, the decoupling isn't perfect anymore at the frequencies f_1 and f_2. But it is still the best at those frequencies; the off-diagonal elements of the decoupled FRF show a dip at f_1 and f_2.

A third problem is caused by the fact that the transformation matrices are not unique anymore. If the system is dyadic, the transformation matrices are independent of the choice of c_1 and c_2. However, in the considered application, the decoupling quality strongly depends on the choice of f_1 and f_2.

It was notified that the decoupling is best at the frequencies f_1 and f_2. When the *interaction index* of the measured FRF is more or less constant within the frequency band of interest, an appropriate choice of f_1 and f_2 will be around 1/3 and 2/3 of the frequency band of interest. This yields acceptable overall decoupling quality within this frequency band. The decoupling becomes poor at frequencies much higher or much lower than f_1 and f_2, but around those frequencies, the decoupling is satisfactory. When the *interaction index* shows a peak at a certain frequency, the decoupling will be most critical at that frequency, so the optimal choice of f_1 and f_2 will be around that frequency. In section 4.1 a method is described to find the optimal f_1 and f_2.

Thanks to these three changes to Owens' DTM-procedure, the procedure is applicable to measured FRF's of a system which is not perfectly dyadic. In the next section, this method will be used to calculate an initial estimate of the optimization procedure which is used to further optimize these transformation matrices.

4 Practical procedure to calculate transformation matrices

In the adapted DTM-procedure (section 3.2), the transformation matrices are determined by the choice of only two parameters, namely f_1 and f_2. In fact, the 4×4 transformation matrices contain 32 parameters. It is obvious that an optimization of all parameters of the transformation matrices can give a better decoupling of the measured FRF. That's why the practical procedure to calculate the best transformation matrices is an optimization of the transformation matrices itself.

The calculation of the initial estimates of the transformation matrices, which are necessary to start the optimization procedure, is described in section 4.1. These initial estimates are based on the method of section 3.2. The final optimal transformation matrices are found with the optimization routine described in section 4.2. The whole procedure is summarized in section 4.3.

4.1 Initial estimate

The only problem left, when applying the adapted DTM-method of section 3.2 to the measured FRF, is the choice of f_1 and f_2. To obtain the optimal choice of f_1 and f_2, a small optimization routine is used. The cost-function which is minimized is:

$$\sum_f \left[\sum_{i=1}^{p} \left(\sum_{j=1}^{p} \left| \left(\tilde{\mathbf{G}}(f) - \mathbf{T_Y}\tilde{\mathbf{D}}_\mathbf{d}(f)\mathbf{T_U} \right)_{ij} \right|^2 \right) \right]. \tag{9}$$

The first summation is a summation over all frequency-lines of the measured FRF. This cost-function is the comparison between the measured FRF, $\tilde{\mathbf{G}}(f)$, and $\mathbf{T_Y}\tilde{\mathbf{D}}_\mathbf{d}(f)\mathbf{T_U}$. Remember that $\tilde{\mathbf{D}}_\mathbf{d}(f)$ is the result of neglecting the off-diagonal elements in $\tilde{\mathbf{D}}(f) = \mathbf{T_Y}^{-1}\tilde{\mathbf{G}}(f)\mathbf{T_U}^{-1}$.

This corresponds to the second way of evaluating the decoupling quality, described in section 2.2. If we assume that very accurate SISO-identification is easily possible (which is validated in section 5), the difference between $\tilde{\mathbf{D}}_\mathbf{d}(f)$ and $\mathbf{D_d}(s)$ is very small. This implies that the minimization of equation (9) corresponds to the optimization of the decoupled description of equation (1).

When the starting-values of f_1 and f_2 are chosen at 1/3 and 2/3 of the frequency band of interest, the optimization is fast and easy. That's why the initial estimate for the main optimization procedure (section 4.2) is based on a small optimization procedure itself: the high accuracy of the initial estimate speeds up the main optimization procedure a lot. As will be seen in the next section, the main optimization procedure optimizes all elements of the transformation matrices. This could become cumbersome if the initial guess is not accurate.

4.2 Optimization procedure

When optimizing the transformation matrices, the 32 parameters of the transformation matrices are not 32 degrees of freedom in the optimization procedure. Multiplying a column of $\mathbf{T_Y}$ by a certain factor, and dividing the corresponding row of $\mathbf{T_U}$ by the same factor gives the same decoupling. This scaling-problem is solved by taking the first elements of every column $\mathbf{T_Y}$ equal to 1, yielding 28 independent parameters left to optimize.

The basic cost-function is the same as used to optimize the choice of $f_1$and f_2 in the previous section (equation (9)). The following adaptations of the cost-function allow the user to do some fine-tuning of the optimization-procedure to get the most useful transformation matrices.

- When the FRF is measured up to high frequencies, we are not interested in perfect decoupling at those frequencies. The same is done during identification, where often an accurate model is desired within the frequency band of interest, but the model-quality outside this band is much less important. That's why a frequency-dependent weighting factor is added to the cost-function.
- As mentioned before, the measured FRF of a four poster has already a certain degree of diagonal dominance. So, in the comparison between the measured FRF and $\mathbf{T_Y}\tilde{\mathbf{D}}_\mathbf{d}(f)\mathbf{T_U}$, it is most important that the diagonal elements fit well. That's why an extra $p \times p$ weighting matrix is added, which multiplies the appropriate elements of the MIMO-FRF by the correct weighting constants. Moreover, the interaction between opposite corners of the car is very small, so these elements of the FRF's can almost be neglected in the optimization procedure.
- It is often interesting to minimize the relative error, instead of the absolute error as in equation (9). Therefore, the cost-function is multiplied by the inverse of the measured FRF. Without this, the decoupling at anti-resonances could be very bad.

The resulting cost function is summarized in the following equation:

$$\sum_{f,i,j} \frac{\left| \left(\tilde{\mathbf{G}}(f) - \mathbf{T_Y}\tilde{\mathbf{D}}_\mathbf{d}(f)\mathbf{T_U} \right)_{ij} \right|}{\left| \tilde{\mathbf{G}}_{ij}(f) \right|} w(f)\mathbf{W}_{ij}. \tag{10}$$

$\mathbf{W}$ is the constant $p \times p$ weighting matrix, to attach more importance to the appropriate elements of the FRF, $w(f)$ is the frequency dependent weighting function to emphasize the frequency band of interest in the cost-function.

To start the minimization of this cost-function with a standard optimization routine, an initial guess of $\mathbf{T_U}$ and $\mathbf{T_Y}$ is required. This is provided by the method described in the previous section. A good initial guess yields fast minimization of the cost-function of equation (10).

4.3 Summary of decoupling procedure

The complete procedure to find an accurate decoupled description of the four poster can be summarized as follows:

1. Measure the MIMO-FRF as accurate as possible.
2. Choose f_1 and f_2 and start the DTM-based minimization procedure of equation (9).
3. The resulting transformation matrices of the previous step are the initial values for the minimization of the cost-function of equation (10).
4. Once the optimal transformation matrices are calculated, $\tilde{\mathbf{D}}(f) = \mathbf{T_Y}^{-1}\tilde{\mathbf{G}}(f)\mathbf{T_U}^{-1}$ can be calculated.
5. The p SISO-identifications of the diagonal elements of $\tilde{\mathbf{D}}(f)$ result in the decoupled model: $\mathbf{D_d}(s) = \ diag\left\{d_{d_1}(s), \ldots, d_{d_p}(s)\right\}$.

5 Simulation and experimental results

5.1 Results on Simulink-model

A simulation model of a four poster is built in a Matlab/Simulink environment. The inputs are the displacements of the hydraulic actuators. On a real four poster every hydraulic actuator is position controlled, mostly by an industrial PID-controller. The dynamics of these control loops are neglected in the simulation model, so the position of the actuator can be directly steered. The simulation model consists of a car-body supported by four suspensions. The car-body is not rigid: torsion and bending of the body is included in the model. The suspensions are modeled as a nonlinear spring and a nonlinear damper, connected with the mass of a wheel. Another spring represents the stiffness of the tyre. This spring is the connection between the actuator and the mass of a wheel. The outputs of the simulation model are the positions of the car-body at the connection points with the suspension. Remark that at a real four poster the outputs are accelerations instead of positions, but this makes no difference for decoupling.

The Simulink-model is used, because the obtained results are much clearer, and better suitable to explain the different aspects of the decoupling than the results of the measured FRF. Although no noise or disturbances are added to the model, the FRF's plotted in this section show some noisy behaviour at higher frequencies. This is due to the modeled nonlinearities in the suspension. The different steps of section 4.3 are now applied to the Simulink-model.

The first step is the measurement of the FRF. It is important to excite the nonlinear system in a similar way as during the durability test to obtain the optimal linear approximation of the nonlinear system. Optimal excitation is obtained with multisines [12]. In [13], it is described how the complete FRF matrix can be measured based on 4 tests. During every test all wheels are excited between 0.1 and 30 Hz. To average out the stochastic nonlinearities, 5 different sets of 4 tests are executed and the average of the 5 measured FRF's, resulting from these sets, is calculated.

The second step is the calculation of the initial estimates of the transformation matrices. The required initial values of f_1 and f_2 are chosen at about 10 and 20 Hz. In the third step, the cost-function (equation (10)) is minimized. Weighting functions were not used in this example. They will become important when the decoupling problem is more difficult, such as in the following section.

After calculation of the decoupled FRF ($\tilde{\mathbf{D}}(f)$) in step 4, the diagonal elements can be identified (step 5). Figure 3 shows the identification result of the fourth diagonal element. A quick non-linear least squares algorithm [12] was used. The model-order selection cannot be based on physical insight, because the decoupled FRF's have no direct physical meaning. Some trial-and-error is necessary. But with some experience, SISO-identification can be very accurate, without taking too much time.

To check the decoupling quality, the interaction index, λ, (section 2.2) is plotted in figure 4. The interaction in the originally measured FRF, is much larger than one at all frequencies. This means that there is no diagonal dominance in the system. The dashed line in figure 4 shows that the decoupling based on the initial estimate of $\mathbf{T_Y}$ and $\mathbf{T_U}$ (section 4.1) is already good. Step 3 of the decoupling procedure gives still a small improvement (dotted line). It is clear that the decoupled system is diagonal dominant, so it can be controlled very well by 4 independent SISO-controllers.

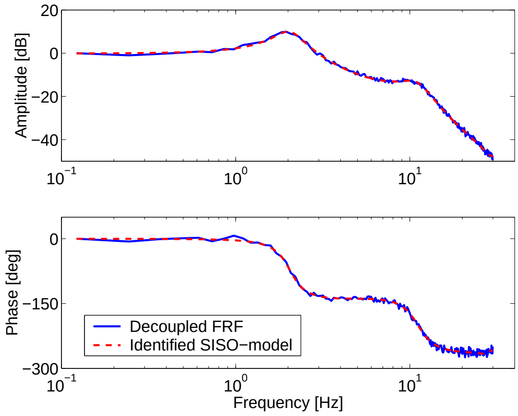

Figure 3: Identification of fourth diagonal element of decoupled FRF: $\tilde{\mathbf{D}}_{44}(f)$ in solid line and the corresponding identified SISO-model ($d_{d_4}(s)$) in dashed line.

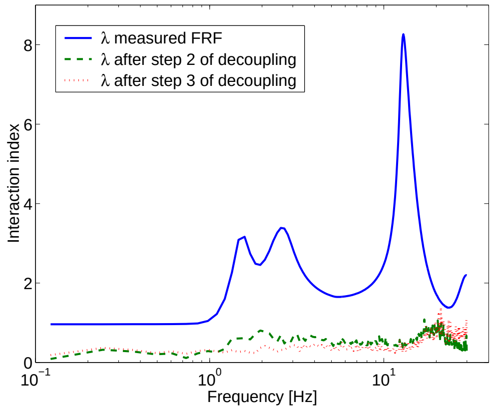

Figure 4: The interaction index of the measured FRF (solid), of the decoupled FRF after step 2 of decoupling procedure (dashed) and based on optimal transformation (dotted).

In figure 5 and 6, the measured FRF is compared with a MIMO-model $\mathbf{G}(s)$ and the decoupled description $\mathbf{T_Y D_d}(s)\mathbf{T_U}$. $\mathbf{G}(s)$ is the result of a subspace algorithm ([3],[4]). The order of the MIMO-model is 10. Higher order models could give a better fit, but then the subspace method resulted in unstable models, which are useless for controller design. Other MIMO-identification methods (Non-linear least-squares method and Maximum-Likelihood method) were tried, but the results are worse. Figure 5 and 6 show the results for the FRF between input 4 and output 4 and between input 2 and output 1 respectively. These are arbitrary choices. The results for the other elements of the measured FRF are comparable. It is clear that the decoupled description is more accurate than the MIMO-model. The MIMO-model cannot fit all (anti-) resonances, especially at higher frequencies.

From these simulation results it is clear that this type of system can be described as in equation (1), at least as accurate as with a MIMO-model. Moreover, the decoupled description offers following advantages: no time-consuming $p \times p$ MIMO-identification is necessary, and robust well-understood SISO-control can be used to control a MIMO-plant.

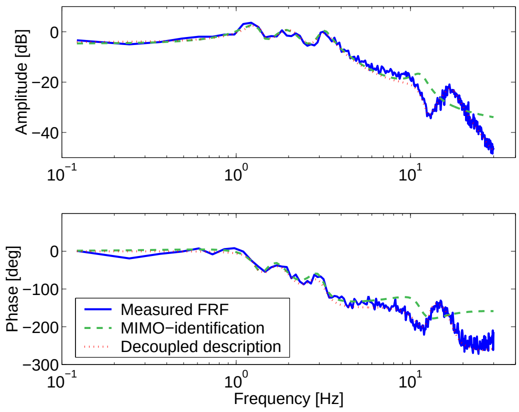

Figure 5: Comparison of the measured FRF of the Simulink-model (solid line), the MIMO-model $\mathbf{G}(s)$ (dashed line) and the decoupled description $\mathbf{T_Y D_d}(s)\mathbf{T_U}$ (dotted line) from input 4 to output 4.

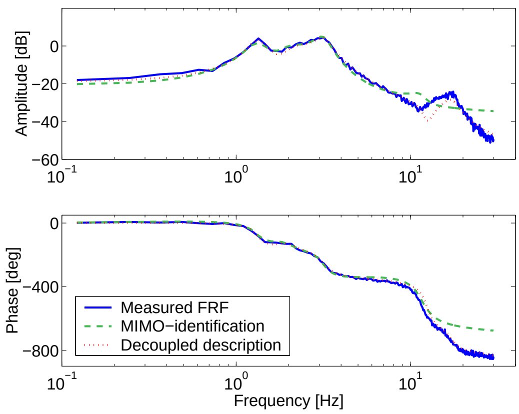

Figure 6: Comparison of the measured FRF of the Simulink-model (solid line), the MIMO-model $\mathbf{G}(s)$ (dashed line) and the decoupled description $\mathbf{T_Y D_d}(s)\mathbf{T_U}$ (dotted line) from input 2 to output 1.

5.2 Results on a four poster

The final validation of the proposed procedure is performed on a measured FRF of a four poster. The inputs are the signals sent to the position-controlled hydraulic actuators, the outputs are the measured accelerations of the chassis near the suspensions. The measured FRF's have quite a noisy behaviour, due to non-linearities in the car, disturbances and measurement noise. Figure 7 and 8 show the results. Once again, the decoupled description is as accurate as the MIMO-model.

The MIMO-identification of $\mathbf{G}(s)$ is very difficult. Again different identification algorithms are tried out, and the best results are plotted. The decoupling procedure was applied straightforward. In step 3, a weighting matrix which emphasizes the diagonal elements gave better results. This could be expected, due to the very noisy behaviour of the off-diagonal elements of the measured FRF.

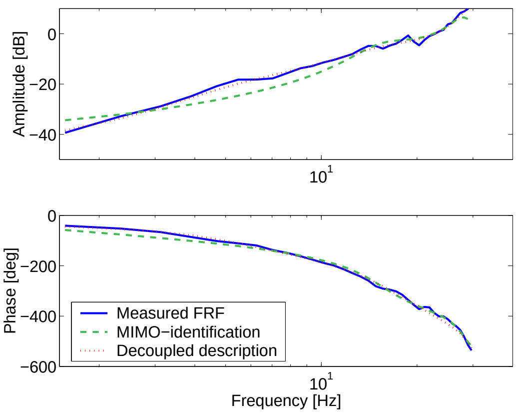

Figure 7: Comparison of the measured FRF of the four poster (solid line), the MIMO-model $\mathbf{G}(s)$ (dashed line) and the decoupled description $\mathbf{T_Y D_d}(s)\mathbf{T_U}$ (dotted line) from input 2 to output 2.

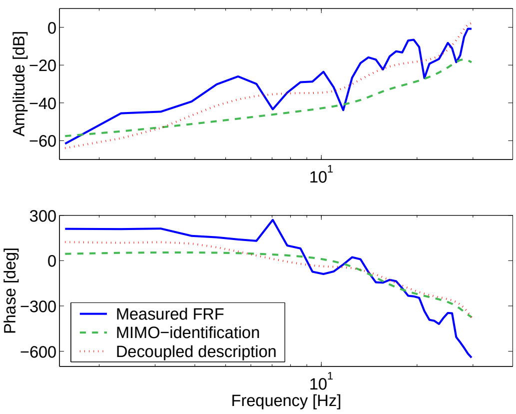

Figure 8: Comparison of the measured FRF of the four poster (solid line), the MIMO-model $\mathbf{G}(s)$ (dashed line) and the decoupled description $\mathbf{T_Y D_d}(s)\mathbf{T_U}$ (dotted line) from input 2 to output 1.

6 Conclusions and further research

This paper describes a procedure to calculate transformation matrices which decouple as good as possible a square MIMO-plant. Using this transformation allows to design a MIMO-controller based on SISO-identification and SISO-controller design methods. It is shown that the decoupled description is at least as accurate as a classical MIMO-model, based on MIMO-identification, if the system has a certain degree of symmetry. Thanks to the decoupling, robust SISO-controllers can be used for MIMO-plants. Nowadays, a lot of MIMO-systems are controlled by decentralized control, because MIMO-identification and MIMO-controller design are very difficult. The interactions are then treated as disturbances for the SISO-controllers. Of course, this puts some limitations on the performance of these controllers. If decoupling is used, the interaction is incorporated in the controller design, without losing the advantage of using SISO-identification and SISO-controller design.

The continuation of this research will validate the proposed control scheme on a four poster. Also other decoupling schemes will be investigated. In section 3.2 it is mentioned that complex transformation matrices decouple perfectly at two frequencies. If such transformation matrices are calculated at every frequency, this results in frequency dependent transformation filters. It will be investigated if this could result in better decoupling, without loosing the simplicity of the decoupling procedure.

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