Full-field modal identification using image moment descriptors

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Abstract
The vibrational characteristics of structures may be determined experimentally by modal testing with piezoelectric accelerometers using well-established methods. However, the placement of accelerometers is critical for complete vibration measurement and unwanted mass-loading effects are introduced. Optical sensing technologies including Scanning Laser Doppler Vibrometry (SLDV), Digital Speckle Pattern Interferometry (DSPI) and Digital Image Correlation (DIC) make full-field non-contact vibration measurement possible. There are numerous benefits of applying these types of optical measurement techniques when measuring, for example, the vibration of rotating, high-temperature, exceedingly small or lightweight structures. However, SLDV measurements are vulnerable to rigid body motion and asynchronism whereas DSPI is only applicable to displacement measurement under resonant, free-free conditions and with limited vibration amplitude. Three dimensional DIC (3D DIC) is a stereophotogrammetric technique which measures displacement on the surfaces of structures. The availability of high speed CMOS cameras enables full-field modal testing using 3D DIC. The full-field measurements are always information redundant but it is possible to apply image processing to extract succinct, efficient and noise-robust features or attributes from the raw data. Modal testing using full-field data is then transformed into system identification using shape features. In this paper, the idea of modal testing using shape features from full-field measurement is illustrated by a case study in the form of a car bonnet of 3D irregular shape. The complex bonnet surface, which is typical of many engineering structures, is essentially a 2-manifold. The 3D surface may be mapped onto a 2D planar parametric domain by the process of surface parameterisation. Then well-developed 2D image processing methods may then be used to extract shape features from full-field displacement of the bonnet. Discrete conformal mapping (DCP) is adopted to determine the two parametric coordinates of the car bonnet. A 2D adaptive geometric moment descriptor (AGMD) defined on the surface parametric coordinates is proposed to extract shape features from the full-field displacement. Gram-Schmidt orthogonalisation is employed to maintain the independence of each AGMD over the measurement domain. The obtained features are shown to be effective and succinct because the raw DIC measurements (more than 14000 spatial data points for each time step) are effectively condensed to approximately 20 AGMD terms.
The frequency response function of shape features (FRF-SF), i.e. the transfer function between the AGMD of the DIC measurement and the AGMD of excitation in this case study, is derived. It is found that the FRF-SF has a similar format to the receptance FRF. The conventional system identification techniques, e.g. curve-fitting, may be applied to determine the modal properties from the experimental FRF-SF. Natural frequencies, damping factors and eigen-shape-feature-vectors are identified from the estimated FRF-SF (using Welch’s averaging method) of the bonnet under random excitation at a single point. The identified natural frequencies are validated by conventional modal testing with accelerometers. The full-field mode shapes reconstructed from eigen-shape-feature-vectors show very good agreement with those predicted by a detailed FE model.

1 Introduction

Modal testing is an experimental approach to determine vibration behavior. The conventional sensors are point-wise piezoelectric accelerometers. Mass loading is one of the major issues when measuring small specimens. The placement of sensors is also critical for obtaining appropriate modes [2]. Measurements on rotating structure and high temperature surfaces are extremely difficult to obtain using adhesively-attached accelerometers. These limitations may be overcome by using non-contact optical sensing technologies which include digital image correlation (DIC) [11], speckle pattern interferometry (SPI) [7, 16], scanning laser Doppler vibrometry (SLDV) [10], automated photoelasticity [6] and thermoelastic stress analysis (TSA) [8] etc.

The three dimensional digital image correlation (3D DIC) method is a stereophotogrammetric technique [12]. The full-field transient response of the specimen’s surface may be measured by employing the high-speed complementary metal oxide semiconductor (CMOS) cameras. The obtained displacement field may be information redundant and noisy [9]. It is desired to condense the full-field data into a set of compact and informative attributes or features. Image decomposition is one of the possible ways to achieve this. Selecting 2D analytic orthogonal functions, e.g. Fourier series, wavelets, Legendre and Zernike polynomials [13], as decomposition bases or kernel functions is the intuitive option. However, orthogonality is satisfied only if the image domain is exactly the same as the definition domain of the analytic functions. In addition, the surfaces of real engineering components are most probably curved. To apply the 2D orthogonal decomposition on the 3D DIC measured transient displacements, two additional approaches are suggested in this paper - namely surface parameterisation [3, 6] and Gram-Schmidt orthogonalisation. Surface parameterisation maps the 3D space coordinates of a surface onto the 2D planner parametric coordinate system. Then the 2D decomposition kernel functions may be defined on the 2D parametric domain. The Gram-Schmidt approach may be applied to regain the orthogonality of the kernel functions over the DIC measured domain. Appropriate construction of kernel functions then results in effective and succinct shape features of the full-field data [13, 14]. Further analysis, e.g. validation of finite element (FE) models[14], may be carried out by using the shape features.

In this paper, modal identification using full-field shape features is presented. The test specimen is an approximately 1.8m×0.8m polyamide car bonnet. Full-field transient response under random excitation were captured by a 3D DIC system. The adaptive geometric moment descriptor (AGMD) of the full-field data is proposed. The results show only a very small number of shape feature terms is sufficient to describe the raw data. Frequency response functions of the shape features (FRF-SF) are proposed. The experimental FRF-SFs are estimated based on the small number of shape features. Modal properties of the specimen are then successfully identified from the estimated FRF-SFs. The reconstructed mode shapes are compared with the ones predicted by a FE model.

In the next section, the experimental setup is described. Then the image moment descriptor of the full-field data is presented and the shape-feature frequency response function is proposed. Results are produced and discussed, and finally conclusions are drawn.
2 Experimental setup

Random vibration of an approximately 1.8m×0.8mm polyamide car bonnet was measured by a pair of high speed CMOS cameras. The specimen as shown in Fig.1 was suspended by two elastic chords to simulate the free boundary conditions. A shaker was connected to the plate by a stinger. Random excitation of 0-128Hz was then applied. Transient images were captured by the two high speed cameras simultaneously in a rate of 300 frames per second. Approximately 24 seconds of images were record (7200 steps). The DIC algorithm was then applied to evaluate the transient displacement from each pair of the stereo-images. Roughly 14158 data points were evaluated at each temporal step. In additional, the force supplied by the shaker was recorded.

3 Image moment descriptor

In this section, a set of orthonormal polynomials defined on the measured surface domain by the 3D DIC system are derived. The deflection shapes as evaluated by the DIC algorithm are then projected onto the orthonormal space spanned by the derived kernel or basis functions to obtain the AGMD.

Suppose \( W(x, t) \) denotes the transient displacement field as measured on the specimen’s surface, it may be expressed as linear combination of a set of kernel or basis functions \( K_i(x) \) defined on the same domain as

\[
W(x, t) = \sum_i a_i(t) K_i(x)
\]  

(1)

where \( a_i, i=0,1,... \) are the combination coefficients, \( x \equiv \{x, y, z\} \in Q \) and \( Q \) denotes the measured surface domain. It is desired to obtain a good approximation of \( W(x, t) \) by only a small number of combination coefficients and expressed as

\[
W(x, t) = \sum_{i=1}^{\mathcal{M}} a_i(t) K_i(x) + e^{\mathcal{M}}(x)
\]  

(2)

where \( \mathcal{M} \) is a positive integer, \( e^{\mathcal{M}}(x) \) is the approximation error. The objective of an efficient decomposition is to determine sufficiently small number of \( \mathcal{M} \) subject to \( \|e^{\mathcal{M}}(x)\|_2^2 < T_a \) where \( T_a \) is a pre-specified
threshold.
Orthogonal decomposition is an intuitive option to select or construct kernel functions. In this case, the decomposition coefficient, called the shape feature or attribute, may be determined by the inner product of \( \mathcal{W}(x, t) \) with \( K_\ell(x) \), expressed as

\[
a_\ell(t) = \int_{Q(x, y, z)} \mathcal{W}(x, t) K_\ell^*(x) \, ds
\]

which is the first kind of surface integral since \( \mathcal{W}(x, t) \) is a scalar function defined as the displacement along the normal direction of the specimen’s surface. The superscript ‘*’ denotes the complex conjugate.

### 3.1 Surface parameterisation

The 3D DIC system captures surface responses of the specimens which are usually non-flat with irregular boundaries in real engineering components. The 3D complex surface is essentially a 2-manifold. It may be possible to apply the 2D image processing approaches based on the 2 parametric coordinates of the 2-manifold. There are numerous techniques of mapping 3D surfaces onto 2D planar domains [3, 5]. This process is usually called surface parameterisation. An example of parameterising a 3D discrete image of Nefertiti’s face is shown in Fig.2. It is seen that the surface is meshed by a set of triangles. The number of grids and triangles should be the same before and after the mapping. This is called a discrete version of the surface parameterisation. The objective is to determine the parametric coordinates \( u \equiv (u, v) \) of the grids subject to the minimum distortion of the triangles transformed from the grids \( x \equiv (x, y, z) \) in 3D space. There are generally three types of distortion measures, namely the angle, area and distance respectively. Apart from the types of distortion, there are other important conditions should be considered – including boundary conditions (fixed or free), bijectivity (invertible or not) and complexity (linear or nonlinear) [5]. The mapping result shown in Fig.2(b) is the discrete conformal mapping (minimum angular distortion of the triangular mesh) with a circular boundary and in Fig.2(c) is the discrete conformal mapping with free boundaries.

### 3.2 Construction of adaptive geometric moment descriptor

The surface parameterisation approaches may be applied to ‘flatten’ the specimen. The parametric coordinates of the specimen’s surface are determined by discrete conformal mapping. The 2-variable functions may then be defined on the obtained parametric coordinates. In particular, the 2D monomial is considered and written as,

\[
g_\ell(u, v) \equiv g_{m,n}(u, v) \equiv u^m v^n
\]

where \( m, n \in \{0, 1, \ldots\} \) are orders of the monomials and \( \ell \in \mathbb{N} \) is the single subscript index. The boundaries of the measured domain are most probably irregular in real engineering applications. To construct an orthonormal version of Eqn.(4) the Gram-Schmidt orthogonalisation (GSO) approach may be employed and expressed as

\[
G_\ell(u, v) \equiv \text{GSO}_\Omega(g_\ell(u, v))
\]

where \( \Omega \) denotes the measured domain in parametric space. Then the orthonormality of Eqn. (5) may be expressed as

\[
\int_\Omega G_i(u, v) G_j(u, v) \, du dv = \delta_{i,j}
\]

where \( \delta_{i,j} \) denotes the Kronecker delta function.
3.3 Adaptive geometric moment descriptor (AGMD)

Shape features of the full-field transient deflection patterns of the specimen may be obtained by adopting the 2D monomials defined in Eqn.(5) as kernel functions. It is denoted the adaptive geometric moment descriptor (AGMD) in this case.

The surface integral of Eqn.(3) may be expressed in parametric form as

\[ a_\ell (t) = \int_\Omega W(x(u, v), t) K_\ell^*(x(u, v)) \mu dudv \]

(7)

where \( \mu \equiv \left\| \frac{\partial x}{\partial u} \times \frac{\partial x}{\partial v} \right\| \) and the real-valued kernel function

\[ K_\ell (x(u, v)) \equiv \tilde{K}_\ell (u, v) = \frac{1}{\sqrt{\mu}} G_\ell (u, v) \]

(8)

Substitution of Eqn.(8) into Eqn.(7) yields,

\[ a_\ell (t) = \int_\Omega W(x(u, v), t) G_\ell (u, v) \sqrt{\mu} dudv \]

(9)

The discrete version of Eqn.(9) defined on the DIC measured grids may be expressed as [15]

\[ a_\ell (t_j) \approx \sum_k W(x(u_k, v_k), t_j) G_\ell (u_k, v_k) \sqrt{\Delta_k^k \Delta_{uv}^k} \]

(10)

where \( \Delta_k^k \) and \( \Delta_{uv}^k \) are the areas of the Voronoi diagram [1] at the \( kth \) grid in the 3D surface and the 2D parametric space, respectively; \( t_j \) is the \( jth \) temporal sample; \( (u_k, v_k) \) are the parametric coordinates at the \( kth \) grid point.
Figure 3: AGMDs of the deflection pattern at 4 temporal samples.

The obtained AGMDs at four temporal samples are shown in Fig.3. The orders of the \( m \) and \( n \) in Eqn(4) were taken to be \( \{0 \sim 10\} \) and \( \{0 \sim 8\} \) respectively. Thus, totally 99 terms of the AGMD were calculated at each temporal step. It is seen from the examples in Fig.3 the AGMDs are sparsely distributed in the lower orders. It is possible to reconstruct the deflection patterns by retaining only the most significant AGMDs so that

\[
\hat{W}(x, t_j) = \sum_{i \in I_M} a_i(t_j) K_i(x)
\]  \hspace{1cm} (11)

where \( I_M \) denotes the set of the most significant AGMDs.

Similarity measure between the original shape pattern and the reconstructed ones from \( M \) terms of the AGMD may be defined as using Pearson’s correlation coefficient

\[
R_j \equiv \frac{\langle W(x, t_j), \hat{W}(x, t_j) \rangle}{\sqrt{\langle W(x, t_j), W(x, t_j) \rangle \langle \hat{W}(x, t_j), \hat{W}(x, t_j) \rangle}} \hspace{1cm} (12)
\]

where\( \langle f(x, t_j), g(x, t_j) \rangle \) denotes inner product, which may be written as

\[
\langle f(x, t_j), g(x, t_j) \rangle \equiv \int_Q f(x, t_j) g(x, t_j) \, ds \hspace{1cm} (13)
\]

Fig.4 shows the values of \( R_j, j=1, \ldots, 7200 \) at all measured steps when retaining only the 20 most significant AGMD terms. It is clear that almost all the correlation coefficients are greater than 99%. Thus the full-field deflection pattern with more than 14k data points at every step are effectively represented by 20 terms of the AGMD. The retained AGMDs correspond to the lower order monomials with longer wavelengths. Thus, the noise is restricted to the short wavelength AGMDs, which are omitted, and has a minor effect on the retained AGMDs. The modal properties of the specimen may then be determined by the frequency responses of the AGMDs.

### 4 Frequency response function of shape features

The frequency response function of the shape features (FRF-SF) may be expressed as,
\( \beta_{\ell,n}(\omega) \equiv \frac{A_{\ell}(\omega)}{B_n(\omega)} \) \hspace{1cm} (14)

\( A_{\ell}(\omega) = \int_{-\infty}^{\infty} a_{\ell}(t) e^{-i\omega t} dt \) \hspace{1cm} (15)

\( B_n(\omega) = \int_{-\infty}^{\infty} b_n(t) e^{-i\omega t} dt \) \hspace{1cm} (16)

where

\( b_n(t) = \int_{Q(x,y,z)} F(x,t) K_n^*(x) ds \) \hspace{1cm} (17)

and \( F(x,t) \) is the full-field pattern of excitation.

It is found that the FRF-SF has similar format as the receptance FRF [15]. For example the FRF-SF for a proportionally damped system may be expressed as [15]

\( \beta_{\ell,n}(\omega) = \sum_{r=1}^{N} \frac{\theta_{r}\theta_{n r}}{\omega_r^2 - \omega^2 + i\omega^2\eta_r} \) \hspace{1cm} (18)

where \( \omega_r \) and \( \eta_r \) are the \( r \)th natural frequency and damping factor, respectively; \( \theta_{\bullet} \) denotes the \( \bullet \)th shape feature of the \( r \)th mode shape. Detailed derivation of Eqn.(18) may be found in [15]. The vector \( \{ \cdots, \theta_{r,k}, \theta_{r,k+1}, \cdots \}^T \) is the \( r \)th eigen-vector of shape features.

## 5 Modal identification

It is seen from the previous section that the transient full-field deflections may effectively be represented by only a small number of AGMDs. Modal identification may be carried out using the AGMDs. The frequency response function of the AGMDs, denoted by FRF-SF, were estimated by Welch’s average method from the measured data. The measurement noise is then reduced by omission of the short wavelength AGMDs. It is seen from Eqn.(18) that the FRF-SF has the same format as the receptance FRF. It is possible to apply the conventional methods to extract the parameters in Eqn.(18). The general curve fitting approach [4] was
applied to determine the natural frequencies, damping factors and residue terms in Eqn.(18). Five modes are identified from 0-128Hz based on the FRF-SF of the 20 most significant AGMDs.

The amplitudes of the FRF of 4 AGMDs are shown in Fig.5. The blue solid and red dashed lines represent tested and fitted by the non-linear least squares approach, respectively. The corresponding shape kernel functions defined on the surface parametric space are shown on the sub-figures as well. It clearly shows that the curves are closely fitted from 6 to 60 Hz. The natural frequencies, damping factors and eigen-shape-feature-vector are then identified from the fitted curves.

The reconstructed mode shapes from the identified eigen-shape-feature-vectors are shown in Fig.6. The corresponding natural frequencies and damping factors are shown on top of the mode shapes.

Different AGMDs may be sensitive to different modes. As shown in Fig.7(a) the 4th and 7th kernel functions showing twisting patterns dominate the mode at 9.2Hz in the range of 6-15Hz. The 1st mode therefore exhibits a twisting pattern - as seen in Fig.6. Similarly, the 6th and 8th AGMDs in Fig.7(b) are only sensitive to the mode at 11.0Hz in the frequency range of 6-15Hz. The 6th and 8th kernel functions are very similar to the 2nd mode as seen in Fig.6.

A FE model of the car bonnet was constructed as well. The FE mode shapes were compared with the experimental estimated from the DIC data. Comparison was carried out based on shape features vectors. The cosine distance of the shape feature vectors between the FE and DIC mode shapes are shown in Fig.8. It is seen that the first 10 FE modes are highly similar to the DIC mode shapes as the cosine distances of the feature vectors are very close to unity.

6 Conclusions

Optical sensing technologies enable full-field non-contact measurements for structural mechanics. The image decomposition approach was applied to reduce the redundancy of information and the measurement noise present in the raw measurement. Orthogonal polynomials are among the most commonly adopted decomposition kernel functions. The advantages of orthogonal decomposition include unique representation and ease of evaluation. The 3D DIC system captures the surface response of the specimen. The surface
Figure 6: Reconstructed mode shapes from identified eigen-shape-feature-vectors as binary colour maps.

Figure 7: Frequency response of AGMDs with respect to shaker force; the digits in brackets on top of the figure are the mode numbers; (a) the 4th and 7th AGMDs; (b) the 6th, 8th and 18th AGMDs.
domain is usually non-flat with irregular boundary conditions in real engineering components. A set of adaptively constructed kernel functions using 2D monomials defined on the measured surface domain was presented. The obtained shape features, called the adaptive geometric moment descriptors, allow effective and succinct representation of the transient full-field data. Shape-feature frequency response functions of the AGMD were developed and modal testing of the specimen under random excitation was carried out based on the AGMDs. Natural frequencies, damping factors and eigen-shape-feature vectors were successfully identified from the experimentally estimated FRF-SF. Vibration mode shapes were then reconstructed from the eigen-shape-feature vectors. The proposed approach was shown to be applicable for full-field modal testing from generally measured data. The advantages of the AGMDs include efficiency of expression, robustness to noise, and ease of estimating the FRF at any point within the measured domain.

References


