Detection, identification, and quantification of sensor fault

J. Kullaa
Aalto University School of Science and Technology, Department of Applied Mechanics
P.O. Box 14300, FIN-00076 Aalto, Finland
email: jyrki.kullaa@metropolia.fi

Abstract
In structural health monitoring (SHM) and control, the structure can be instrumented with a redundant sensor network, which can be utilized in sensor fault diagnosis. In this study, the objective is to detect, identify, and quantify a sensor fault using the structural response data from the sensor network. Seven different sensor fault types are investigated and modelled: bias, gain, drifting, precision degradation, complete failure, noise, and constant with noise. Each sensor in the network is modelled using the minimum mean square error (MMSE) estimation and the sensor fault is identified and quantified using the multiple hypothesis test utilizing the generalized likelihood ratio (GLR). The proposed approach is experimentally verified with a sensor network assembled on a wooden bridge. Different sensor faults are simulated by modifying a single sensor. The method is able to detect a sensor fault, identify and correct the faulty sensor, as well as identify and quantify the fault type.

1 Introduction
With emerging sensor technology, an increasing number of sensors can be installed to structures for monitoring and control. Due to the high number of low-cost sensors, sensor faults become more frequent compared to the structure’s lifetime. As the monitoring or control applications utilize the sensor data for decision, it is important that the data acquired are accurate and reliable. A faulty sensor cannot perform its function properly but instead may provide false information for decision, thus making the system unreliable. Therefore, it is necessary to detect such failures and adapt to the new situation with correcting actions. With a high number of sensors, the system is redundant, and removing a sensor will not result in a loss of information. This fact can be utilized in detecting, isolating and correcting a faulty sensor. Sometimes, it is also important to identify the type and magnitude of sensor fault, for example to find the origin of the fault or to design more robust sensors. In this paper, an automatic method is proposed to detect and isolate the faulty sensor in a network, and to identify the type and magnitude of the fault.

A brief review of sensor validation research is given in the following, which is by no means exhaustive. There are two main approaches: 1) hardware redundancy and 2) analytical redundancy [1]. The first approach uses the fact that several sensors measure the same quantity. The second approach utilizes a mathematical model of the system, for example a finite element model, and the redundancy is provided by the model. Both approaches have their advantages and disadvantages. For hardware redundancy, extra sensors are required, and for analytical redundancy, an accurate mathematical model must be created. We restrict here to hardware redundancy.

The analysis can be based on the parity space approach which uses a measurement model $y = Hx$ to estimate the sensor network. More specifically, the anomalies in the data are found from the null-space of $H$. Here $y$ is a vector of the measurements (sensors), $H$ is the measurement matrix and $x$ is a vector of the unknown state variables. For a redundant system, the number of columns in $H$ must be higher than the number of rows. The disadvantage of this approach is that the measurement matrix $H$ is often unknown. One could try to identify it or alternatively use a direct statistical method in which the aforementioned measurement model is not explicitly used.
Sensor validation has been studied e.g. in [2, 3] using the parity space approach. Other methods include principal component analysis (PCA) [4–9], independent component analysis (ICA) [9], factor analysis [10], and minimum mean square estimation (MMSE) [11, 12]. Comparisons of different methods have been performed in [9, 10].

There are few studies identifying the types or magnitudes of sensor fault. The objective of this paper is to identify the most common sensor faults and assess the severity of the fault, or more specifically, estimate the fault parameters of the sensor.

This paper is organized as follows. The sensor model is presented in section 2. This model is then used in section 3 to detect sensor fault with a composite hypothesis test using the generalized likelihood ratio test (GLRT). Different sensor fault models are derived in section 4. An algorithm is proposed in section 5 to identify and quantify the sensor fault using the fault models and the multiple hypothesis test. Experimental results are presented in section 6 to validate the proposed method. Finally, concluding remarks are given in section 7.

2 Sensor model

The proposed model is a conditional probability density function for each sensor given the other sensors in the network. It is assumed that the sensor network has enough redundancy so that a data-based approach can be used, without a need of a physics-based (e.g. finite element) model. In vibration-based monitoring, the sensor network typically consists of accelerometers or strain gauges. If the number of the sensors in the network is higher than the number of active natural modes, the network is redundant [5, 6].

The assumptions are that the sensor network is redundant, all sensors are simultaneously sampled, and that there are training data from the sensor network with all sensors functioning. The data can be time signals or static features. In the applications, we restrict to accelerations at different locations of the structure. Also no physics-based model is used. No restriction is made about the stationarity of the process, because samples at the same time instant (spatial correlation) are only used in this study. It is also assumed that at most one sensor is faulty and that the fault exists during the whole measurement. The faulty sensor, fault type, fault magnitude, or the measurement having a faulty sensor are not known in advance.

The sensor model is derived by estimating each sensor using the others in the network. This can be done by applying the minimum mean square error (MMSE) estimation. The model is identified using the training data from the functioning sensor network. The MMSE model is briefly introduced in the following.

The sensors are divided into observed sensors \( v \) and missing sensors \( u \):

\[
y = \begin{bmatrix} u \\ v \end{bmatrix}
\]

(1)

with a partitioned covariance matrix \( \Sigma \) of the training data

\[
\Sigma = \begin{bmatrix} \Sigma_{uu} & \Sigma_{uv} \\ \Sigma_{vu} & \Sigma_{vv} \end{bmatrix} = \begin{bmatrix} \Gamma_{uu} & \Gamma_{uv} \\ \Gamma_{vu} & \Gamma_{vv} \end{bmatrix}^{-1}
\]

(2)

where the precision matrix \( \Gamma \) is defined as the inverse of the covariance matrix \( \Sigma \) and is also written in the partitioned form. A linear MMSE estimate for \( u|v \) (u given v) is [12]:

\[
\hat{u} = E(u|v) = \mu_u - \Gamma_{uv}^{-1}(v - \mu_v)
\]

(3)

where \( K = -\Gamma_{uv}^{-1}\Gamma_{vv}, \mu_u \) and \( \mu_v \) are the mean of \( u \) and \( v \), respectively, and \( E(\cdot) \) is the expectation. The error covariance matrix (MSE) is
\[ C = \text{cov}(u|v) = \Gamma^{-1} \]  \hspace{1cm} (4)

Notice that in most references, for example in [13], Equations 3 and 4 are given in a different form using the covariance matrix instead of the precision matrix. The proposed form, however, results in a more efficient algorithm [12].

Assuming a Gaussian distribution, the conditional probability density function (pdf) of \( vu \) can be constructed from the model parameters \( K, C, \mu_u, \) and \( \mu_v \) in Equations 3 and 4:

\[
p(u|v) = \frac{1}{(2\pi)^{p/2} \det(C)^{1/2}} \exp \left[ -\frac{1}{2} (u - \hat{u})^T C^{-1} (u - \hat{u}) \right]
\]  \hspace{1cm} (5)

where \( p \) is the number of sensors to be estimated (dimensionality of \( u \)), which is typically one. Notice that the sensor model was derived without using a physics-based model or an identified parametric model of the system.

The application of the pdf (Equation 5) and the log-likelihood ratio to sensor fault detection, identification, quantification, and reconstruction is discussed in the following sections.

3 GLRT for Sensor fault detection and isolation

The first step in sensor validation is to detect sensor fault. Fault detection is done using a hypothesis test for the MMSE parameters. The hypothesis test used for fault detection is

\[
H_0: \quad K = K_0, \quad C = C_0, \quad \mu = \mu_0 \\
H_1: \quad K \neq K_0, \quad C \neq C_0, \quad \mu \neq \mu_0
\]  \hspace{1cm} (6)

where \( K_0, C_0, \) and \( \mu_0 \) are estimated from the training data. This is a composite hypothesis test with unknown parameters in hypothesis \( H_1 \). For this case, fault detection is performed with the generalized likelihood ratio test (GLRT) [14, 15]. The test statistic is the log-likelihood ratio for each sample:

\[
s = \ln \frac{p(u|v; H_1)}{p(u|v; H_0)}
\]  \hspace{1cm} (7)

where \( p(u|v; H_i) \) is the probability according to hypothesis \( H_i, i = 0, 1 \). Hypothesis \( H_0 \) is that the parameters are equal to those of the training data (normal), and hypothesis \( H_1 \) is that the parameters are different to those of the training data (anomaly). Distribution \( p(u|v; H_1) \) is obtained by estimating the parameters from the current measurement, while the parameters of \( p(u|v; H_0) \) are estimated from the training data. The threshold needed to accept or reject the null hypothesis is discussed in the following.

Each sensor yields one test statistic. In total, the number of variables equals the number of sensors in the network. The dimensionality can be reduced by applying principal component analysis (PCA) [16] to the log-likelihood ratios \( s \) (Equation 7). The variable used for sensor fault detection is finally the subgroup sample standard deviation \( \hat{S} \) of the first principal component scores, which is plotted on an \( \hat{S} \) control chart [17] with appropriate control limits. In this study, the subgroup consists of 4 subsequent variables.

In order to assess the detection performance, the ROC accuracy measure is utilized. A ROC curve (Receiver Operating Characteristic) plots the probability of detection against the probability of false alarm. The area below the ROC curve (ROC accuracy [18]) is a measure of classification performance of two labelled data sets. If the value is one, the two sets are fully separable. If the ROC accuracy is 0.5, the detection is by chance only. In most cases, the value lies between those two extremes, implying that the two data sets are partially overlapping.

Once a sensor fault is detected, the faulty sensor must be found. Faulty sensor isolation is a multiple hypothesis test. The highest rms of the log-likelihood ratio is thus expected to reveal the faulty sensor. The faulty sensor can be corrected by simply replacing the faulty sensor with its estimated value.
4 Sensor fault models

To identify the fault type and magnitude, the following approach is proposed. It is assumed that the faulty sensor has been isolated. The next step is to model different fault types and study the compatibility of the data with each fault model. This is also a multiple hypothesis test.

In the following, models for different sensor faults are derived. They are given in a more general multivariable form, useful if several sensors fail simultaneously or if temporal correlation is also utilized. The models will have a simpler mathematical form in a univariate case, but are otherwise equivalent to those being presented. The maximum likelihood estimates (MLE) of the fault parameters are also given. The number of estimated fault parameters, \( m \), is used in penalizing complex models. For simplicity, the conditional notation \( u | v \) is replaced with \( u \). Variable \( u \) should therefore be understood conditioned with \( v \).

4.1 No fault

If the sensor has no fault, the data model is

\[
\mathbf{u}[n] = \hat{\mathbf{u}}[n] + \mathbf{w}[n]
\]

where \( \hat{\mathbf{u}} \) is given by Equation 3 with parameters \( \mathbf{K}_0 \) and \( \mathbf{\mu}_0 \), and \( \mathbf{w} \) is noise with covariance matrix \( \mathbf{C}_0 \) and \( n \) is the sample number. The conditional pdf is

\[
p(\mathbf{u}[n]) = \frac{1}{(2\pi)^{\frac{\nu}{2}} (\det \mathbf{C}_0)^{\frac{\nu}{2}}} \exp \left[ -\frac{1}{2} (\mathbf{u}[n] - \hat{\mathbf{u}}[n])^T \mathbf{C}_0^{-1} (\mathbf{u}[n] - \hat{\mathbf{u}}[n]) \right]
\]

and the log-likelihood is

\[
\ln p(\mathbf{u}[n]) = -\frac{\nu}{2} \ln(2\pi) - \frac{1}{2} \ln(\det \mathbf{C}_0) - \frac{1}{2} (\mathbf{u}[n] - \hat{\mathbf{u}}[n])^T \mathbf{C}_0^{-1} (\mathbf{u}[n] - \hat{\mathbf{u}}[n])
\]

The number of fault parameters estimated is \( m = 0 \).

4.2 Bias

Bias gives values that are shifted by a constant from the true value. The bias model is

\[
\mathbf{u}[n] = \mathbf{A} + \hat{\mathbf{u}}[n] + \mathbf{w}[n]
\]

where \( \mathbf{A} \) is the unknown bias, the MLE of which is

\[
\hat{\mathbf{A}} = \frac{1}{N} \sum_{n=0}^{N-1} (\mathbf{u}[n] - \hat{\mathbf{u}}[n])
\]

where \( N \) is the number of samples. The log-likelihood is

\[
\ln p(\mathbf{u}[n]) = -\frac{\nu}{2} \ln(2\pi) - \frac{1}{2} \ln(\det \mathbf{C}_0) - \frac{1}{2} (\mathbf{u}[n] - \hat{\mathbf{u}}[n])^T \mathbf{C}_0^{-1} (\mathbf{u}[n] - \hat{\mathbf{u}}[n])
\]

The number of fault parameters estimated is \( m = p \).

4.3 Drifting

The drifting model is
\[ u[n] = A + Bn + \hat{u}[n] + w[n] \]  

where \( A \) and \( B \) are unknown constants. The MLEs of \( A \) and \( B \) are found by maximizing

\[
p(\mathbf{u}; A, B) = \frac{1}{(2\pi)^{\frac{N}{2}} (\det C_0)^{\frac{N}{2}}} \exp \left[ -\frac{1}{2} \sum_{n=0}^{N-1} (u[n] - A - Bn - \hat{u}[n])^T C_0^{-1} (u[n] - A - Bn - \hat{u}[n]) \right]
\]

with respect to \( A \) and \( B \). This is equivalent to maximizing

\[
-\frac{1}{2} \sum_{n=0}^{N-1} (u[n] - \hat{u}[n] - A - Bn)^T C_0^{-1} (u[n] - \hat{u}[n] - A - Bn)
\]

resulting in

\[
\begin{pmatrix}
\sum_{n=0}^{N-1} n^2 & \sum_{n=0}^{N-1} n \\
\sum_{n=0}^{N-1} n & N
\end{pmatrix} \begin{bmatrix}
A_i \\
B_i
\end{bmatrix} = \begin{bmatrix}
\sum_{n=0}^{N-1} (u_i[n] - \hat{u}_i[n]) \\
\sum_{n=0}^{N-1} n (u_i[n] - \hat{u}_i[n])
\end{bmatrix}
\]

where \( A_i, B_i, \) and \( u_i \) are the \( i \)th components of vectors \( A, B, \) and \( u \), respectively. From [19], p. 42:

\[
\begin{pmatrix}
N & N(N-1) \\
N(N-1) & 2N(N^2-1)
\end{pmatrix}
\]

Finally, the estimates of \( A_i \) and \( B_i \), are computed from

\[
\begin{pmatrix}
\hat{A}_i \\
\hat{B}_i
\end{pmatrix} = \begin{pmatrix}
\frac{2(2N-1)}{N(N+1)} & -\frac{6}{N(N+1)} \\
-\frac{6}{N(N+1)} & \frac{12}{N(N^2-1)}
\end{pmatrix} \begin{pmatrix}
\sum_{n=0}^{N-1} (u_i[n] - \hat{u}_i[n]) \\
\sum_{n=0}^{N-1} n (u_i[n] - \hat{u}_i[n])
\end{pmatrix}
\]

The log-likelihood is

\[
\ln p(\mathbf{u}[n]) = -\frac{P}{2} \ln(2\pi) - \frac{1}{2} \ln(\det C_0) - \frac{1}{2} \left( u[n] - \hat{A} - \hat{B}n - \hat{u}[n] \right)^T C_0^{-1} \left( u[n] - \hat{A} - \hat{B}n - \hat{u}[n] \right)
\]

The number of fault parameters estimated is \( m = 2p \).

### 4.4 Precision degradation

The precision degradation model is

\[ u[n] = \hat{u}[n] + s[n] + w[n] \]  

where \( s[n] \) is a Gaussian random process with zero mean and an unknown covariance matrix \( C_s \). Because \( s[n] \) and \( w[n] \) are independent, the total covariance is \( C = C_0 + C_s \). The MLE of \( C \) is obtained by

\[
\hat{C} = \frac{1}{N} \sum_{n=0}^{N-1} (u[n] - \hat{u}[n])(u[n] - \hat{u}[n])^T
\]

from which \( \hat{C}_s = \hat{C} - C_0 \). The log-likelihood is
\[ \ln p(u[n]) = -\frac{p}{2} \ln(2\pi) - \frac{1}{2} \ln(\det \tilde{C}) - \frac{1}{2} \left( u[n] - \tilde{u}[n] \right)^T \tilde{C}^{-1} \left( u[n] - \tilde{u}[n] \right) \]  

(23)

The number of estimated fault parameters is

\[ m = \frac{p(p+1)}{2} \]  

(24)

4.5 Gain

Gain is a multiplicative fault and the sensor model is

\[ u[n] = G(\tilde{u}[n] + w[n]) \]  

(25)

where \( G \) is an unknown gain (constant). Notice that we restrict here to the univariate case. The pdf is

\[ p(u[n]) = \frac{1}{(2\pi\sigma_0^2)^{1/2}} \exp \left[ -\frac{1}{2\sigma_0^2} \left( u[n] - G\tilde{u}[n] \right)^2 \right] \]  

(26)

in which variance \( \sigma_0^2 \) is given by Equation 4. The MLE of \( G \) is found by maximizing the log-likelihood

\[ \ln p(u; G) = -\frac{N}{2} \ln(2\pi\sigma_0^2) - N \ln(G) - \frac{1}{2G^2\sigma_0^2} \sum_{n=0}^{N-1} \left( u[n] - G\tilde{u}[n] \right)^2 \]  

(27)

Taking the derivative of the log-likelihood function with respect to \( G \) and setting the result equal to zero, produces

\[ \hat{G}^2 + \hat{G} \frac{1}{N\sigma_0^2} \sum_{n=0}^{N-1} u[n]\tilde{u}[n] - \frac{1}{N\sigma_0^2} \sum_{n=0}^{N-1} u[n]^2 = 0 \]  

(28)

or

\[ \hat{G}^2 + b\hat{G} + c = 0 \]  

(29)

where

\[ b = \frac{1}{N\sigma_0^2} \sum_{n=0}^{N-1} u[n]\tilde{u}[n] \]  

(30)

\[ c = -\frac{1}{N\sigma_0^2} \sum_{n=0}^{N-1} u[n]^2 \]

Solving for \( G \) produces two solutions

\[ \hat{G} = \frac{1}{2} \left[ -b \pm \sqrt{b^2 - 4c} \right] \]  

(31)

from which the larger root is selected (assuming \( G > 0 \)). The log-likelihood is

\[ \ln p(u[n]) = -\frac{p}{2} \ln(2\pi) - \ln \hat{G} - \frac{1}{2} \ln \sigma_0^2 - \frac{1}{2G^2\sigma_0^2} \left( u[n] - \hat{G}\tilde{u}[n] \right)^2 \]  

(32)

The number of parameters estimated is \( m = 1 \).

4.6 Complete failure 1: Constant

In the first complete failure, the sensor gives a constant value. The sensor model is
\( \mathbf{u}[n] = \mathbf{A} \)  

where \( \mathbf{A} \) is an unknown constant (DC level). Because the variance is zero, the pdf equals 1 if the sensor reading is \( \mathbf{A} \). Otherwise the pdf equals zero. This type of distribution is computationally inconvenient, as the covariance matrix is singular. Therefore, a diagonal covariance matrix \( \mathbf{C} \) is used instead with small positive values. Constant \( \mathbf{A} \) is estimated as before:

\[
\hat{\mathbf{A}} = \frac{1}{N} \sum_{n=0}^{N-1} \mathbf{u}[n] 
\]

(34)

The log-likelihood is

\[
\ln p(\mathbf{u}[n]) = -\frac{p}{2} \ln(2\pi) - \frac{1}{2} \ln(\det \mathbf{C}) - \frac{1}{2} (\mathbf{u}[n] - \hat{\mathbf{A}})^\top \mathbf{C}^{-1} (\mathbf{u}[n] - \hat{\mathbf{A}}) 
\]

(35)

The number of parameters estimated is \( m = 1 \).

### 4.7 Complete failure 2: Constant + noise

In the second case of complete failure, the sensor gives a constant value and bottom noise. The conditional sensor model is

\[
\mathbf{u}[n] = \mathbf{A} + \mathbf{s}[n] 
\]

(36)

where \( \mathbf{A} \) is an unknown constant (DC level) and \( \mathbf{s} \) is noise with an unknown covariance matrix \( \mathbf{C}_s \). The MLE of \( \mathbf{A} \) is computed with Equation 34 and the MLE of \( \mathbf{C}_s \) is

\[
\hat{\mathbf{C}}_s = \frac{1}{N} \sum_{n=0}^{N-1} (\mathbf{u}[n] - \hat{\mathbf{A}})(\mathbf{u}[n] - \hat{\mathbf{A}})^\top 
\]

(37)

The log-likelihood is

\[
\ln p(\mathbf{u}[n]) = -\frac{p}{2} \ln(2\pi) - \frac{1}{2} \ln(\det \hat{\mathbf{C}}_s) - \frac{1}{2} (\mathbf{u}[n] - \hat{\mathbf{A}})^\top \hat{\mathbf{C}}_s^{-1} (\mathbf{u}[n] - \hat{\mathbf{A}}) 
\]

(38)

The number of parameters estimated is

\[
m = p + \frac{p(p + 1)}{2} 
\]

(39)

### 4.8 Complete failure 3: Bottom noise

In the third case of complete failure, the sensor gives bottom noise. The sensor model is

\[
\mathbf{u}[n] = \mathbf{s}[n] 
\]

(40)

where \( \mathbf{s} \) is noise with an unknown covariance matrix \( \mathbf{C}_s \). The MLE of \( \mathbf{C}_s \) is

\[
\hat{\mathbf{C}}_s = \frac{1}{N} \sum_{n=0}^{N-1} \mathbf{u}[n] \mathbf{u}[n]^\top 
\]

(41)

The log-likelihood is

\[
\ln p(\mathbf{u}[n]) = -\frac{p}{2} \ln(2\pi) - \frac{1}{2} \ln(\det \hat{\mathbf{C}}_s) - \frac{1}{2} \mathbf{u}[n]^\top \hat{\mathbf{C}}_s^{-1} \mathbf{u}[n] 
\]

(42)

The number of parameters estimated is
5 Sensor fault identification and quantification

Once the faulty sensor has been isolated, the next step is to identify the fault type and magnitude. To this end, the multiple hypothesis test is applied. Each fault type is fitted to the faulty sensor with the ML estimates of the fault parameters. An additional term $-\frac{m}{2} \ln N$ is added to penalize models having a large number of parameters [15]. The hypothesis $H_i$ is chosen which maximizes $L_i$ defined by

$$L_i = \sum_{n=0}^{N-1} s_i[n] - \frac{m_i}{2} \ln N = \sum_{n=0}^{N-1} [\ln p(u[n]; H_i) - p(u[n]; H_0)] - \frac{m_i}{2} \ln N$$  \hspace{1cm} (44)$$

where $p(u[n]; H_i)$ is the probability according to hypothesis $H_i$ (fault type $i$), $i = 0 \ldots 7$ and $m_i$ is the number of model parameters of fault type $i$. The log-likelihood ratio has a nice normalization property, i.e. if the sensor is not faulty the log-likelihood ratio is close to zero. The fault magnitude is given by the estimated parameters of the identified fault type.

Sensor fault identification and quantification are performed using the following procedure.

1. Define the training data with no sensor faults.
2. Estimate mean $\mu$ and covariance matrix $\Sigma$ and compute the precision matrix $\Gamma = \Sigma^{-1}$ of the training data.
3. Set the faulty sensor as a missing variable $\mathbf{u}$.
4. Form matrices $\mathbf{u}_0, \mathbf{u}, \mathbf{G}_{uv}, \mathbf{G}_{vu}$ by partitioning $\mathbf{u}$ and $\mathbf{\Gamma}$.
5. Compute the error covariance matrix using Equation 4.
6. Compute the mean of $\mathbf{u}|\mathbf{v}$ for each sample using Equation 3.
7. Take next measurement.
8. Select fault type $i$.
9. Estimate parameters of fault type $i$ for the current measurement (hypothesis $H_i$).
10. Compute $\ln p(u[n]; H_i)$ for fault type $i$.
11. Compute $L_i$ for fault type $i$ using Equation 44.
12. Return to 8 until all fault types have been evaluated.
13. Return to 7 until all measurements have been evaluated.
14. Decide measurement, fault type $i$, and fault magnitude (fault parameters) corresponding to maximum $L_i$.

6 Experimental research

An experimental research was performed with a monitoring system built in the laboratory. The structure was a wooden bridge model shown in figure 1. Random excitation was applied to the structure to excite the lowest modes. Fifteen accelerometers measured the response at three different longitudinal positions. The sampling frequency was 256 Hz and the measurement period was 32 s. For sufficient redundancy, the data were low-pass filtered below 64 Hz and re-sampled.

The data used in this study were four measurements, each having 4076 samples. The training data were samples 5001–10000, and the in-control data for control chart design were samples 1–5000. Sensor fault was introduced to the last measurement (samples 12229–16304). Fault detection was performed by plotting the $S$ control chart for the first principal component of the log-likelihood ratio. The area below the ROC curve was used as a measure to assess the detectability of the fault.

Different sensor faults were simulated in each sensor with five different magnitudes. The fault magnitude was a multiple of the sensor’s standard deviation $\sigma$ in the training data, except for gain. The fault cases are listed in table 1. It should be noted that with a complete failure, the fault magnitude has little meaning.
### Table 1: Parameter $k$ for different sensor fault types and magnitudes

<table>
<thead>
<tr>
<th>Fault type</th>
<th>Symbol</th>
<th>Parameters</th>
<th>XS</th>
<th>S</th>
<th>M</th>
<th>L</th>
<th>XL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bias</td>
<td>B</td>
<td>$A = k\sigma$</td>
<td>0.01</td>
<td>0.0422</td>
<td>0.1778</td>
<td>0.7499</td>
<td>3.1623</td>
</tr>
<tr>
<td>Drifting</td>
<td>D</td>
<td>$A = k\sigma$</td>
<td>0</td>
<td>0.2500</td>
<td>0.5000</td>
<td>0.7500</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$A + BN = k\sigma$</td>
<td>0.05</td>
<td>0.6625</td>
<td>1.2750</td>
<td>1.8875</td>
<td>2.5</td>
</tr>
<tr>
<td>Precision degradation</td>
<td>PD</td>
<td>$C = (k\sigma)^2$</td>
<td>0.01</td>
<td>0.0422</td>
<td>0.1778</td>
<td>0.7499</td>
<td>3.1623</td>
</tr>
<tr>
<td>Gain</td>
<td>G</td>
<td>$G = k$</td>
<td>0.9</td>
<td>1.2</td>
<td>0.7</td>
<td>1.4</td>
<td>0.5</td>
</tr>
<tr>
<td>Constant</td>
<td>C</td>
<td>$A = k\sigma$</td>
<td>0.001</td>
<td>0.01</td>
<td>0.1</td>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>Constant + noise</td>
<td>CN</td>
<td>$A = k\sigma$</td>
<td>0.01</td>
<td>0.0422</td>
<td>0.1778</td>
<td>0.7499</td>
<td>3.1623</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$C = (k\sigma)^2$</td>
<td>0.01</td>
<td>0.0422</td>
<td>0.1778</td>
<td>0.7499</td>
<td>3.1623</td>
</tr>
<tr>
<td>Noise</td>
<td>N</td>
<td>$C = (k\sigma)^2$</td>
<td>0.01</td>
<td>0.0422</td>
<td>0.1778</td>
<td>0.7499</td>
<td>3.1623</td>
</tr>
</tbody>
</table>

### 6.1 Gain in sensor 3

As a first example, consider a gain fault in sensor 3 with a small magnitude of $G = 0.9$ during measurement 4. Figure 2 shows the results of the different fault diagnosis steps. Anomaly detection from the sensor network is shown in figure 2a, where a change in the system is clearly visible. The area below the ROC curve was 0.997 implying a high classification performance. The next step, faulty sensor isolation, is depicted in figure 2b where the log-likelihood ratio for each sensor is plotted, showing that the faulty sensor was correctly identified having the largest value. Fault identification and quantification of sensor 3 was performed for each measurement and fault type. The result is presented in figure 2c where the mean $L_i$ (Equation 44) of each measurement and fault type is depicted. The fault types are represented by bars with different colours. The fault type was correctly identified as a gain fault in measurement 4 with magnitude $\hat{G} = 0.9007$, which is very close to the true value. Finally, the faulty sensor was estimated, and a detail of the reconstructed sensor together with the true sensor without fault is plotted in figure 2d. The two curves are hardly distinguishable implying an accurate reconstruction of sensor 3.
6.2 Bias in sensor 9

As a second example, consider a bias fault in sensor 9 with a medium magnitude of $A = k\sigma = 0.0080$ during measurement 4. Figure 3 shows the results of the different fault diagnosis steps. Anomaly detection from the sensor network is shown in figure 3a, where a change in the system is clearly visible. The area below the ROC curve was 0.991, implying that the data from the faulty sensor are separable from those of the functioning sensor. However, as can be seen, the control limit is frequently exceeded in the in-control range suggesting that wider limits should be used. The next step, faulty sensor isolation is depicted in figure 3b, showing that the faulty sensor was correctly identified. Fault identification and quantification of sensor 9 was performed for each measurement and fault types. The result is presented in figure 3c. The fault type was correctly identified as a bias fault in measurement 4 with magnitude $A = k\sigma = 0.0081$, which is very close to the true value. Drifting fault would have been chosen if no penalty term for model complexity had been used. The estimated parameters for drifting were $A = k\sigma = 0.0082$ and $A + BN = k\sigma = 0.0080$ implying that the value of parameter $B$ was very small. Finally, the faulty sensor was estimated, and a detail of the reconstructed sensor together with the true sensor without fault is plotted in figure 3d. Reconstruction of sensor 9 was less accurate than that of sensor 3 in the previous example. This was due to the fact that MSE of sensor 9 was higher that that of sensor 3.
Figure 3: Bias $0.0080$ in sensor 9. a) Detection, b) isolation, c) quantification, and d) reconstruction (detail). The red dashed line is the true sensor without fault and the blue solid line is the estimated sensor.

6.3 Extensive study

An extensive research was performed, in which all faults shown in table 1 were introduced to each sensor in turn. The anticipated results were: 1) detectability of different fault types and magnitudes; 2) faulty sensor isolation performance; 3) identification of different fault types; and 4) quantification of fault magnitudes.

The ROC accuracy averaged over all sensors for each fault type is depicted in figure 4 left as a function of the fault magnitude. The fault magnitude clearly affects the detectability, as expected. For complete failure, the notion of fault magnitude is not meaningful, and the ROC accuracy is therefore almost invariable.

The effect of sensor location on the detectability was also studied. The mean ROC accuracy averaged over all fault types and magnitudes is plotted in figure 4 right for each sensor. Also the minimum and maximum mean ROC accuracies averaged over all fault types are shown. It can be seen that sensors 1–6 had a higher ROC accuracy than sensors 7–15. This effect can be explained by the different sensor MSE values in the sensor network, which is discussed in more detail below.
Figure 4: Left: Overall detectability of different faults with different magnitudes. Right: Detectability of faults in different sensors. Solid line: mean ROC accuracy averaged over all fault types and magnitudes. Dashed lines: maximum and minimum mean ROC accuracies averaged over all fault types.

<table>
<thead>
<tr>
<th>Fault type</th>
<th>XS</th>
<th>S</th>
<th>M</th>
<th>L</th>
<th>XL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bias</td>
<td>4 (3)</td>
<td>15 (6)</td>
<td>15 (11)</td>
<td>15 (15)</td>
<td>15 (15)</td>
</tr>
<tr>
<td>Drifting</td>
<td>6 (4)</td>
<td>15 (15)</td>
<td>15 (15)</td>
<td>15 (15)</td>
<td>15 (15)</td>
</tr>
<tr>
<td>Precision degradation</td>
<td>5 (1)</td>
<td>8 (1)</td>
<td>14 (6)</td>
<td>15 (15)</td>
<td>15 (15)</td>
</tr>
<tr>
<td>Gain</td>
<td>15 (6)</td>
<td>15 (14)</td>
<td>15 (15)</td>
<td>15 (15)</td>
<td>15 (15)</td>
</tr>
<tr>
<td>Constant</td>
<td>15 (15)</td>
<td>15 (15)</td>
<td>15 (15)</td>
<td>15 (15)</td>
<td>15 (15)</td>
</tr>
<tr>
<td>Constant + noise</td>
<td>15 (15)</td>
<td>15 (15)</td>
<td>15 (15)</td>
<td>15 (15)</td>
<td>15 (15)</td>
</tr>
<tr>
<td>Noise</td>
<td>15 (15)</td>
<td>15 (15)</td>
<td>14 (15)</td>
<td>15 (15)</td>
<td>14 (15)</td>
</tr>
<tr>
<td>No fault</td>
<td>10 (-)</td>
<td>10 (-)</td>
<td>10 (-)</td>
<td>10 (-)</td>
<td>10 (-)</td>
</tr>
</tbody>
</table>

Table 2: Number of correct identifications in the network having 15 accelerometers from which one sensor in turn is faulty. The first figure is the number of successful fault type identifications if the faulty sensor is correctly found. The figure in the parentheses is the number of correct isolations of the faulty sensor.

Table 2 shows the number of successful fault type identifications for each fault type and magnitude assuming the faulty sensor was correctly isolated. The number of successful isolations of the faulty sensor is shown in parentheses in the same table. In all cases, the identification of the fault type was more reliable than the isolation of the faulty sensor. This is probably due to the fact that in the fault type identification, a true fault model was used, whereas in the faulty sensor isolation, the sensor model for hypothesis $H_1$ was not always correct. The fault magnitude was observed to have an influence on either result.

Finally, the estimated fault parameters are depicted in figure 5 for each fault type with fault magnitude $M$ (see table 1). Also shown are the true parameter values. The estimated fault magnitudes were quite accurate, except for precision degradation in some sensors. The fault quantification accuracy was generally higher for sensors 1–6 than for sensors 7–15. This can be explained with the last plot in figure 5, showing the MSE for each sensor in the network. The sensor estimation accuracy of sensors 1–6 was much higher than that for sensors 7–15.
7 Conclusion

A generalized likelihood ratio approach was applied to sensor validation. The objective was to detect, isolate, identify, quantify, and correct a sensor fault in a sensor network. The method is based on hardware redundancy, in which several sensors measure the same quantity. The approach was solely based on the measurement data, no physics-based model was used. The method is supposed to be valuable for sensor networks with a large number of low-cost sensors installed to the plant.

Seven different sensor fault types were modelled. Multiple hypothesis test was utilized to identify the fault type and magnitude. A penalty term was used to favour fault types with a small number of parameters. For example, if no penalty term were used, drifting would always be chosen in the case of a bias fault. An alternative to the penalty term would be to check if some of the estimated fault parameters are close to zero.

The sensor locations in the structure was seen to have an effect on the estimation error, which in turn yielded different fault diagnosis performance for different sensors. Therefore, in designing the number and positions of the sensors, MSE should be minimized and designed to be of the same order of magnitude for each sensor. If some sensors have a much higher MSE than the others, it may be an indication of insufficient redundancy.

Sensor fault was assumed to be the sole change in the system. No attempts were made to identify changes due to environmental or operational effects or structural damage. A research to distinguish between different sources of variability is under way.

The identification of the MMSE model requires a large number of time synchronized samples from the sensor network. In order to apply the proposed method in wireless sensor networks (WSN), the energy-efficiency may become an issue. This subject is left for future research.

Figure 5: Fault quantification in different sensors with different faults of magnitude M. The horizontal lines represent the true values of the fault parameters \( k \) (table 1). The last plot shows the MSE of each sensor in the network.
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References
